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Lithium-ion batteries Remaining Useful Life Prediction Method Considering Recovery Phenomenon

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Estimation of lithium battery remaining useful life (RUL) is the key to lithium battery health management. In the process of intermittent discharging lithium batteries, the recovery phenomenon will have a relatively large impact on the life of lithium batteries. However, in the nowadays research on the RUL of lithium batteries, the recovery phenomenon in the process of intermittent discharging lithium batteries is rarely taken into account. In this paper, a degradation model and RUL prediction method of lithium batteries considering the recovery effect are proposed. Firstly, under the framework of Wiener process theory, the RUL degradation model without recovery effect is established. Then, by considering the effect of lithium battery recovery on remaining useful life, the RUL degradation model with recovery effect is derived. And in the sense of the first passage time, theoretically, the RUL distribution of lithium batteries with recovery effects or no recovery effects is derived. Furthermore, the unknown parameters of the model are estimated by the method of maximum likelihood estimation. Finally, the designed experiment obtained the degradation data of 18650 lithium battery, and the method in this paper was applied to verify the effectiveness of the proposed method.

Keywords: remaining useful life ; recovery phenomenon; lithium batteries; degradation modeling

1. INTRODUCTION

As the global energy and environmental crisis has become more serious, the development and expansion of new energy has become the mainstream of energy development strategies. Electric vehicles have become a hotspot in the field of automobile transportation because of their outstanding advantages, such as energy saving, low noise, low emission and energy configurability [1,2]. The development of electric vehicles has the dual benefits of energy conservation and environmental protection. In particular, pure electric vehicles have obvious advantages in total energy conversion efficiency and total pollution emissions, which are the development direction of the future automobile industry [3]. As the energy source of electric vehicles, lithium-ion batteries have the advantages of high voltage, long cycle life, high specific energy and high specific power. Therefore, they are widely used in many fields, and they are the core of the electric vehicle industry chain [4,5].

In recent years, Prognostic and Health Management (PHM) has become a hot issue of research. As the core of PHM, RUL prediction has received extensive attention from industry and academia, and a large number of research results have emerged [6-8]. With the deepening of the research on RUL prediction, many scholars have begun to pay attention to the prediction of the RUL of lithium batteries in recent years. Li et al. [9] proposed an efficient method for estimating RUL online using Support Vector Machine (SVM) algorithm. By studying the characteristics of the lithium-ion battery degradation process, the rising terminal voltage and the variation of the derivative voltage (DV) during the charging process are introduced as training variables of the SVM algorithm to define the battery RUL. Finally, the SVM is used to build a lithium-ion battery degradation model and predict the lithium-ion battery actual cycle numbers. Wang et al. [10] proposed a run-time Reconfigurable Computing (RC) system. The system is on Field Programmable Gate Array (FPGA) for Relevance Vector Machine (RVM) to realize real-time Remaining Useful Life (RUL) estimation. Duong et al. [11] introduced a heuristic Kalman filter algorithm, a meta-heuristic optimization method, combined with particle filtering to deal with sample degradation and predict the RUL of lithium batteries; Ng et al. [12] studied battery degradation under different usage conditions and ambient temperature modeling. Based on a prediction of the RUL of lithium-ion batteries under constant operating conditions and at different ambient temperature, a naive Bayesian model is proposed for the RUL of lithium batteries which is used to forecast the RUL of lithium-ion batteries under different operating conditions. Zhao et al. [13] used particle filter to study the prediction of the remaining service life of lithium-ion batteries, and proposed a simple and effective fusion model method and data-driven method for RUL prediction. The algorithm uses the model and data-driven fusion method to modify the double-exponential empirical degradation model to reduce the difficulty of model parameters and parameter training, and it uses the particle filtering algorithm to track the degradation process of battery capacity and uses the automatic regression model to modify the observations of the state space Equation as well as improve prediction accuracy. Wang et al. [14] proposed a prediction method which is based on the discrete wavelet transform (DWT). The dynamic stress test (DST) schedule of the commercial 1665130 lithium-ion battery was used to obtain and analyze various outputs with non-stationary and transient phenomena. Yang et al. [15] improved PSO algorithm with ELM algorithm to predict lithium-ion battery RUL. Yang et al. [16] used the heuristic Kalman algorithm (HKA) to optimize the input weights and biases of the ELM algorithm. The mean square error (MSE) obtained from the ELM is used as the cost function of the HKA algorithm, and the optimized particles in the HKA are used as the weights and biases of the ELM predictor.

During the charge and discharge cycle, lithium batteries are affected by factors such as temperature, self-discharge rate, depth of discharge, and discharge rate, and their capacity and life are continuously attenuated. When the battery capacity drops to 80% of the initial capacity, the performance of the battery will not meet the power supply technical requirements, and maintenance or replacement is required. Otherwise it will cause serious harm to electric vehicles and personal safety. Therefore, it is necessary to analyze the RUL of the lithium battery in order to use the lithium battery more reasonably and ensure safety.

When using a lithium battery every day, the lithium battery is left for a period of time after use, and it will be found that the remaining capacity of the lithium battery has a certain increase compared with the previous one, that is, there is a phenomenon of " electricity returning", which is the recovery of the lithium battery. Obviously, the recovery phenomenon of lithium battery will affect the life of lithium battery. Therefore, a more general and practical situation in the field of RUL prediction is how to establish a degradation model under the influence of recovery of lithium battery. Integrating the recovery effect into the prediction of the RUL distribution of lithium batteries is a real and urgent need in the life prediction and health management of lithium batteries.

In the method of RUL prediction, from the perspective of economic and safety, the method of RUL prediction based on degradation modeling has become the mainstream [17-19]. Literature [19] systematically and completely reviewed the degradation modeling methods, such as Wiener process, Gamma process, Markov chain and hidden Markov process. However, the existing degradation process described by the Gamma process, Markov chain, and Hidden Markov process basically assumes that the degradation process is monotonous and irreversible. Moreover, in engineering practice, due to the load condition of the equipment, the dynamic change of the internal state, and the change of the external environment, it is possible to accelerate or decelerate the degradation process, that is, the measured performance degradation data has non-monotonic characteristics. The Wiener process has good mathematical properties, and it is suitable for describing non-monotonic random degradation processes with increasing or decreasing trends in engineering practice. At the same time, it can obtain the RUL distribution required by health management, rather than a single point estimation, so it has been widely applied to degradation modeling and RUL estimation [20-22].

In the life cycle of lithium batteries, the recovery phenomenon of lithium batteries will have an impact on the life of lithium batteries. In this paper, with the aim to resolve the problem of lithium battery RUL prediction with recovery effect, a method based on Wiener process degradation modeling and prediction method is proposed. First of all, under the framework of Wiener process theory, the degradation path model in the stage of lithium battery degradation process is established. Secondly, using the nature of the Wiener process, the analytical solution of the RUL of the lithium battery without considering the recovery effect is derived, and the lithium battery affected by recovery is derived. And then, the unknown parameters of the model are estimated by the method of maximum likelihood estimation. Finally, the process of obtaining lithium battery degradation data is introduced, and the method proposed in this paper is verified by using the degradation data of lithium battery.

2. MODELING BASED ON WIENER PROCESS

The device for obtaining the data set used in this paper is the Battery Test System (BTS). The physical map of the device is shown in Fig 1, and the device is composed of three parts. Part 1 is generally a computer, and its function is sending commands to collect and storing the acquired battery data collected by the part 3 in real time; the function of part 2 is realizing the network connection, receive the control command of part 1 , control part 3 , and transmit the real-time data; the function of part 3 is receiving the command of part 2 , control the charging and discharging of the channel, and collect the voltage, current and other data of the channel in real time. The lithium battery used in the experiment is detailed in Table 1.

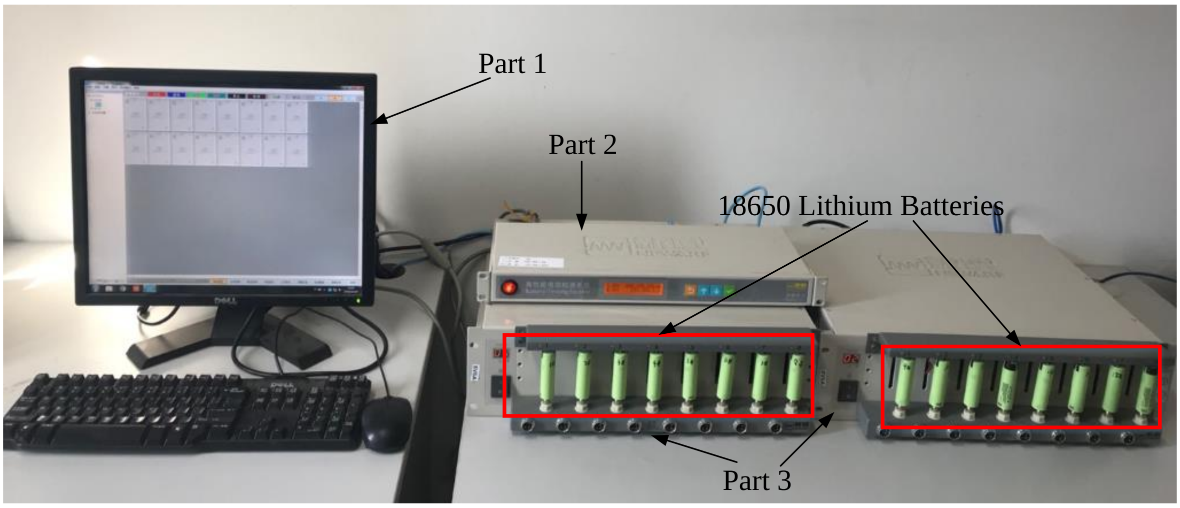


Figure 1. BTS physical map

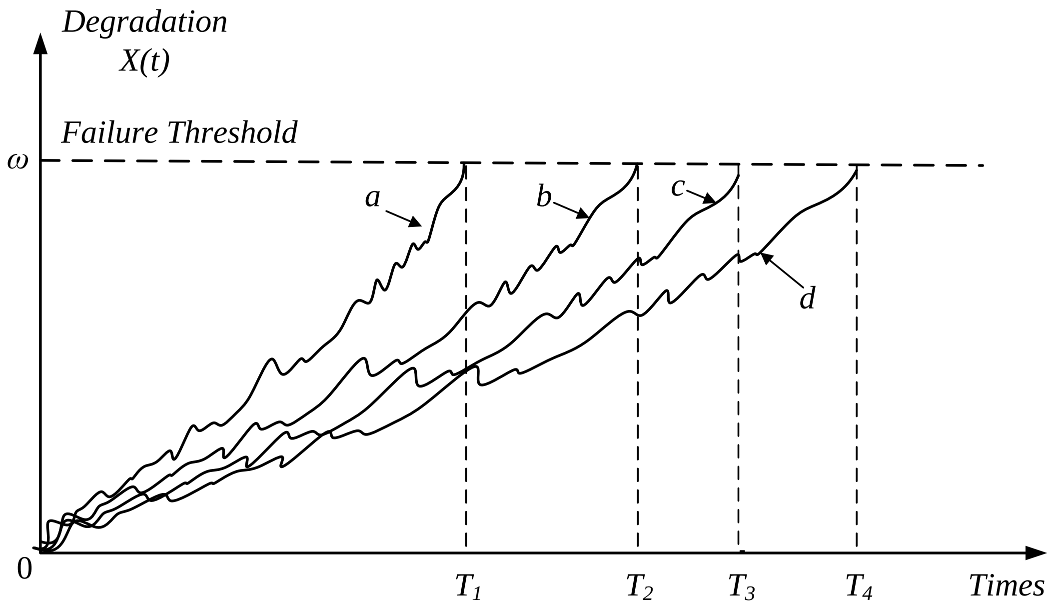


Figure 2. Schematic diagram of the lithium battery RUL considering recovery effect

Based on the raw data of lithium battery degradation, the problem of predicting the lithium battery RUL under the influence of recovery can be described by Fig 2. Where, the abscissa is time t , and the ordinate is the amount of deterioration of the lithium battery $\{X(t), t \geq 0\}$ changing according to time t , ω is the failure threshold of the lithium battery and is generally determined by industry standards, reliability and accuracy requirements of the product. The curves a, b, c, d are respectively the degradation paths under the influence of different recovery time, respectively. T_1、T_2、$T_3 \cdots$ are the lithium battery invalid times under the influence of different recovery time, that is, the life of the lithium battery under different recovery time.

Table 1. Lithium battery details

Items	Specification
Battery Type	NCR18650B
Capacity	3400mAh
Nominal Voltage	3.6V
Charging Voltage	4.2V
Discharging End Voltage	2.5V
Internal Resistance	Less than 100mΩ
Size	Length 65mm; Diameter 18mm
Weight	Less than 48.5g

Based on the first passage time concept of the stochastic process [23], the phase time T of the stochastic process $X(t)$ is defined as the time when the degraded amount of lithium battery reaches a certain preset threshold ω for the first passage time, which can be expressed as follows:

$$T = \inf\left\{t : X(t) \geq \omega \mid X(0) < \omega\right\} \tag{1}$$

Where ω is a preset threshold.

2.1. Modeling of lithium battery degradation process without recovery effect

The degradation model based on Wiener process is a typical linear stochastic model describing the stochastic degradation process of equipment. Such models are widely used in the degradation of mechanical wear and equipment corrosion. In general, the degradation process of a lithium battery can be approximated as a linear process, so a degradation model based on the Wiener process can be used. In general, a degradation model based on the Wiener process can be described as follows:

$$X(t) = X(0) + \theta t + \sigma B(t) \tag{2}$$

Where $X(0)$ is the initial state of the lithium battery, θ is the drift coefficient, σ is the diffusion coefficient, $B(t)$ is the standard Brownian motion, and $\sigma B(t) \sim N(0, \sigma^2)$ is used to characterize the random dynamics of the degradation process. Since the degradation process of a lithium battery can be approximated as a linear process, the degradation model based on the Wiener process is suitable for the degradation process of a lithium battery.

2.2. Modeling of lithium battery degradation process with recovery effect

In the process of degradation of lithium batteries, recovery phenomenon continue throughout. Since the amount of recovery is small each time the recovery phenomenon occurs, for the convenience of observation, the total recovery amount in the entire life cycle is represented by Z, and Z obeys a normal distribution. Therefore, the lithium battery degradation model based on the Wiener process with recovery effects can be described as follows:

$$Y(t) = X(t) + Z \tag{3}$$

Where $Y(t)$ is total degradation amount of the battery at time t. Z is the amount of recovery in the life cycle of a lithium battery, and $Z \sim N(\mu_1, \sigma_1{}^2)$.

3. RUL PREDICTION BASED ON LITHIUM BATTERY RECOVERY

3.1. RUL prediction of lithium battery without recovery effect

First, the case of no recovery effect is considered. In the case of no recovery effect, the degradation model used for lithium battery life prediction is Equation (2), so the probability density function (PDF) and cumulative distribution function (CDF) based on the Wiener process in the first passage time sense can be given by the following theorem.

Theorem 1. [24-26]. According to the definition of Equation (1), the first passage time distribution of the Wiener process is an inverse Gaussian distribution, and the PDF and CDF are respectively as follows:

$$f_T(t) = \frac{\omega - X(0)}{\sigma\sqrt{2\pi t^3}} \exp\left\{\frac{\left[\omega - X(0) - \theta t\right]^2}{2t\sigma^2}\right\} \tag{4}$$

$$F_T(t) = 1 - \Phi\left(\frac{\omega - \theta t}{\sigma\sqrt{t}}\right) + \exp\left(\frac{2\theta\omega}{\sigma^2}\right)\Phi\left(\frac{-\omega - \theta t}{\sigma\sqrt{t}}\right) \tag{5}$$

The expectation and variance of the inverse Gaussian distribution are as follows:

$$E(T) = \frac{\omega - X(0)}{\theta} \tag{6}$$

$$Var(T) = \frac{\omega\sigma^2}{\theta^3} \tag{7}$$

Where ω is pre-set lithium battery failure threshold.

3.2 RUL prediction of lithium battery with recovery effect

Considering the recovery effect, the degradation model used for lithium battery life prediction is Equation (3), so the probability density function (PDF) and cumulative distribution function (CDF) based on the Wiener process in the first passage time sense. which can be given by the following theorem.

Theorem 2. According to the definition of Equation (1), the first passage time distribution of the Wiener process is an inverse Gaussian distribution, and the PDF and CDF are respectively as follows:

$$f_T(t) = \frac{\omega - X(0) - \mu_1}{\sqrt{2\pi\left(\sigma_1^2 + \sigma^2 t\right)^3}} \exp\left\{\frac{\left[\omega - X(0) - \mu_1 - \theta t\right]^2}{2\left(\sigma_1^2 + \sigma^2 t\right)}\right\} \tag{8}$$

$$F_T(t) = 1 - \Phi\left[\frac{\omega - X(0) - \mu_1 - \theta t}{\sqrt{\sigma_1^2 + \sigma^2 t}}\right] + \exp\left\{\frac{2\theta\left[\omega - X(0) - \mu_1\right]}{\sigma^2} + \frac{2\theta^2\sigma_1^2}{\sigma^4}\right\}\Phi\left[\frac{-\omega - \theta t + X(0) + \mu_1 - \frac{2\theta\sigma_1^2}{\sigma^2}}{\sqrt{\sigma_1^2 + \sigma^2 t}}\right]$$

(9)

The expectation and variance of the inverse Gaussian distribution are as follows:

$$E(T) = \frac{\omega - X(0) - \mu_1}{\theta} \tag{10}$$

$$Var(T) = \frac{\theta\sigma_1^2 + \left[\omega - X(0) - \mu_1\right]\sigma^2}{\theta^3} \tag{11}$$

Where ω is pre-set lithium battery failure threshold.

4. PARAMETER ESTIMATION

The parameters to be estimated in this paper include two parts, one is θ, σ in the Equation 2, and the other is the unknown parameter α in the Equation 3.

4.1 Estimation of $\theta, \mu_1, \sigma_1, \sigma$

In order to complete the estimation of the unknown parameters $\theta, \mu_1, \sigma_1, \sigma$ in the Equation 2, it is assumed that there are N devices under test, and the sampling time point of the n th device under test is $t_1^n, \cdots, t_m^n$, where m represents the measured value of the n th device, and $n = 1, \cdots, N$. Therefore, based on the no recovery effect model the degradation path of the k th sample point t_k^n of the n th device can be written as follows:

$$Y^n(t_k) = X^n(0) + \theta t_k^n + \sigma B(t_k) + Z \tag{12}$$

Where $k = 1, \cdots, m$. Quote the intermediate variable $R^n(t_k^n) = X^n(t_k^n) - X^n(0)$, the specific implementation of $R^n(t_k^n)$ is $r^n(t_k^n)$, and the corresponding degradation data is $\left\{R^n(t_k^n) = r^n(t_k^n), n = 1, \cdots, N, k = 1, \cdots, m\right\}$. So, the Equation (12) can be expressed as follows:

$$R^n(t_k^n) = \theta t_k^n + \sigma B(t_k^n) + Z \tag{13}$$

Further, let $T^n = \left(t_k^n, \cdots, t_m^n\right)'$, $R^n = \left(r^n(t_k^n), \cdots, r^n(t_m^n)\right)'$, where $(\bullet)'$ represents the transpose of a vector, and R represents a collection of degradation data at this stage, consisting of $R^n, n = 1, \cdots, N$. According to the independent incremental properties of Equation (13) and the standard Brownian Motion (BM) process, it can be seen that R^n obeys the multidimensional normal distribution, and its mean and covariance are as follows:

$$\mu^n = \theta T^n + \mu_1 \tag{14}$$

$$\Omega^n = \sigma^2 Q^n + \sigma^2 I_m \tag{15}$$

Where

$$Q^n = \begin{bmatrix} t_1^n & t_1^n & \cdots & t_1^n \\ t_1^n & t_2^n & \cdots & t_2^n \\ \vdots & \vdots & \vdots & \vdots \\ t_1^n & t_2^n & \cdots & t_m^n \end{bmatrix} \tag{16}$$

Then, the log likelihood function of θ, σ corresponding to all test numbers R is as follows:

$$\ln L = -\frac{mN}{2}\ln(2\pi) - \frac{N}{2}\ln\left|\Omega^n\right| - \frac{1}{2}\sum_{n=1}^{N}\left(R^n - \mu^n\right)'\left(\Omega^n\right)^{-1}\left(R^n - \mu^n\right) \tag{17}$$

Substituting the Equation (14), the Equation (15) and the Equation (16) into the Equation (17), by maximizing the Equation (17), the MATLAB multidimensional search method can be used to obtain θ and σ. The similar method of maximum likelihood estimation is detailed in the literature [17, 21].

5. EXPERIMENTAL VERIFICATION

In order to verify the effectiveness of the proposed method in this paper, this section predicts the RUL of the 18650 lithium-ion battery commonly used in the new energy field.

5.1 Description of lithium battery degradation process

Lithium-ion battery is the core component of electric vehicles that are currently developing in the field of new energy. It is the power system of electric vehicles and plays a vital role in electric vehicles. Therefore, once the capacity of the lithium battery drops seriously, it will seriously affect the working state of the lithium battery, so that the lithium battery has the danger of spontaneous combustion and explosion, which threatens people's personal and property safety. In the process of actually using a lithium battery, the lithium battery has three stages of charging, discharging, and standing. After the lithium battery is charged, the state of the lithium battery is repeatedly switched between discharging and standing. When the lithium battery is left standing, the battery will recharge. Due to the existence of different lithium ion concentration which is between the positive and negative electrodes of the battery, the lithium ion of the negative electrode of the battery will drive to the negative electrode of the battery due to the difference in concentration. This process is the reverse of the battery discharge, similar to the battery charging process, and the battery's charge state will be improved. For lithium batteries for vehicles, recovery is more common because the car is in a state of driving and stagnation for most of the time. The intensity of recovery is related to the time when the battery is left to rest and changes as the number of times when the intermittent discharging of battery increase, as well as affects the remaining battery life. As times of charge and discharge of the lithium battery increase, the health of the lithium battery (SOH) will decrease. When the SOH of the lithium battery is lower than a certain value, that is, the degradation amount of the lithium battery exceeds the preset failure threshold, it indicates that the lithium battery cannot be used. If it can’t meet the requirements of normal use, it is considered to have terminated its life.

5.2 Acquisition of degradation data

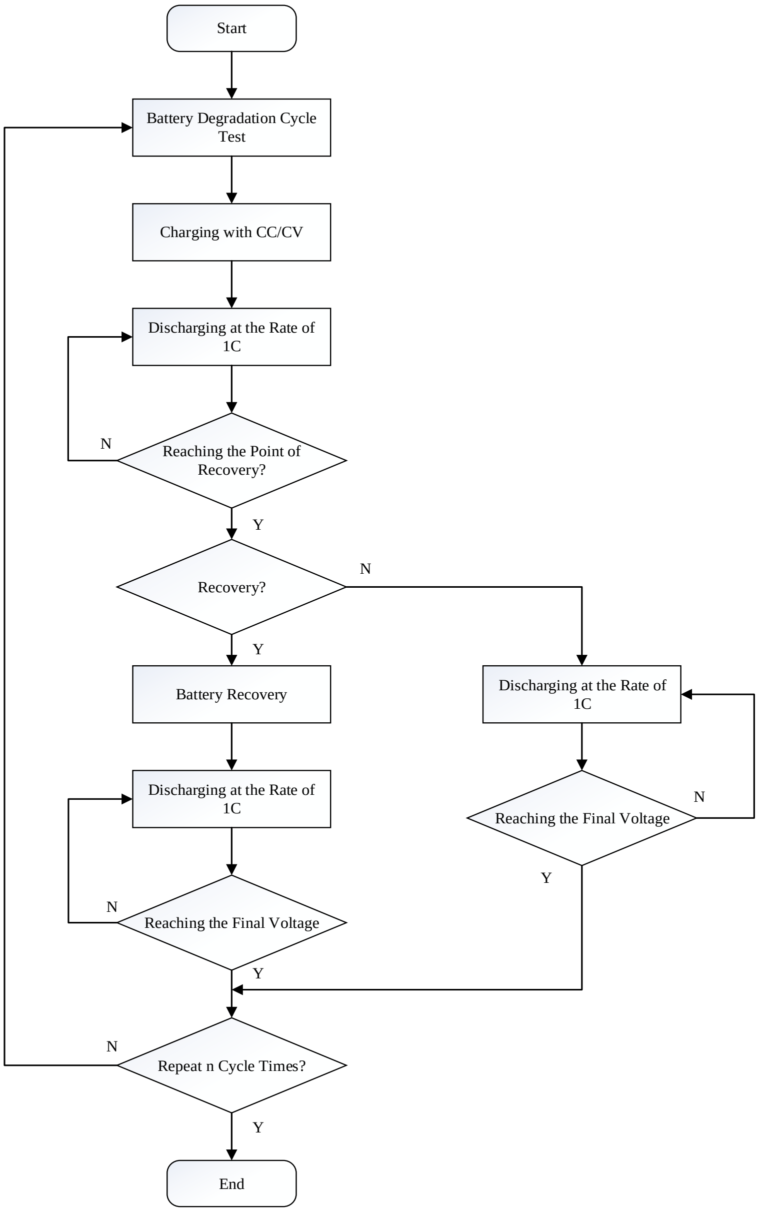


Figure 3. Flow chart of raw data acquisition of lithium battery

The experiments of this paper used BTS to obtain degradation data for lithium batteries. The process of obtaining lithium battery degradation data is shown in Fig 3. The lithium battery is first charged by constant current. When the voltage reaches a certain value, it becomes constant voltage charging, and the charging current is gradually reduced. After the end of charging, the discharge starts at a rate of 1 C. When the voltage of the lithium battery drops to the point where recovery begins, the lithium battery is left on, and the lithium battery begins to heal itself. After recovery, it continues to discharge at a rate of 1 C until the lithium battery voltage drops to the end voltage value. Performing n times of charge and discharge cycle until the degradation amount of the lithium battery reaches the failure threshold.

5.3 Degradation Data of Lithium Batteries in Three Recovery States

The lithium battery degradation data used in this experiment is the monitoring data of the same batch and the same type of lithium battery at the same detecting point, specifically the remaining capacity of the lithium battery. A complete charge and discharge are used as a monitoring point. During the process of obtaining lithium battery degradation data, the test environment (temperature, humidity, etc.) and the energy working time meet the test requirements, making the degradation process relatively stable. The default failure threshold of the lithium battery degradation process is ω =0.2(20%), that is, when the deterioration of the lithium battery capacity exceeds 20% of the rated capacity, the lithium battery fails. In this paper, the first 200 data of two sets of degradation data were used. Unit 1 was used to estimate parameters, and Unit 2 was used for experimental verification. The specific degradation path is shown in Fig 4.

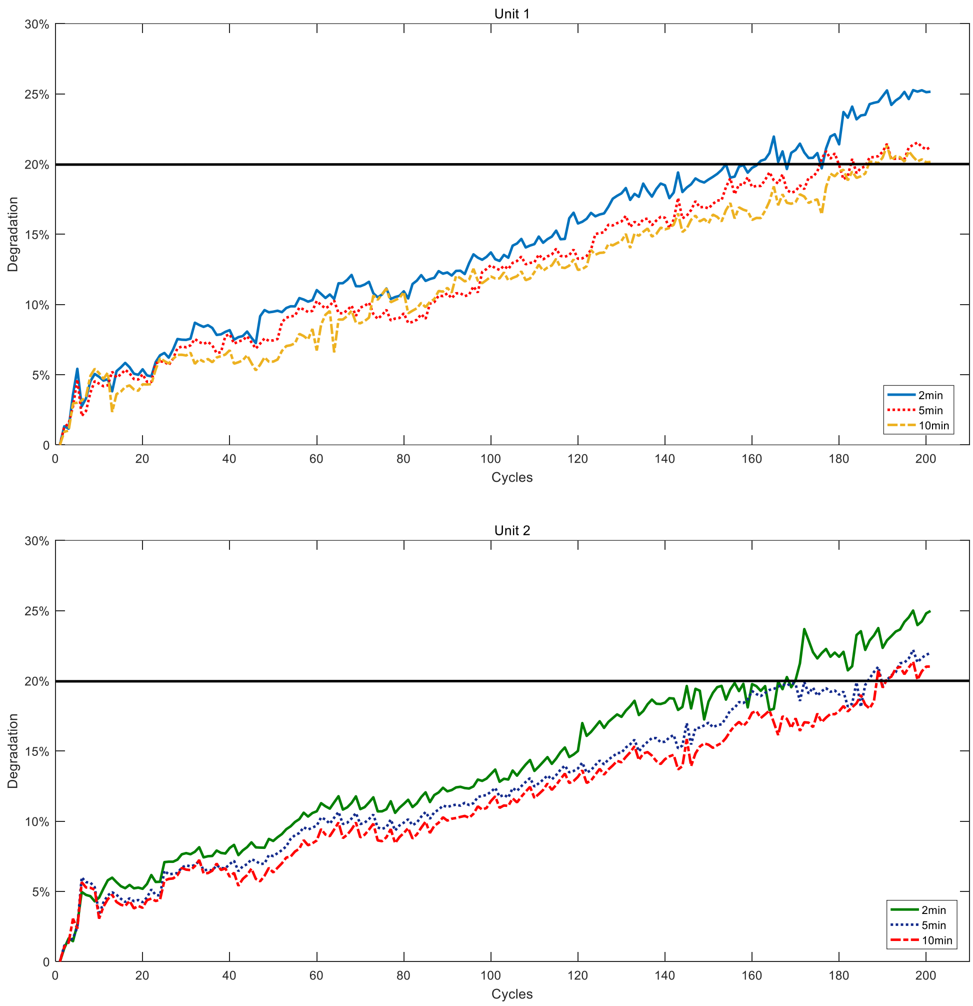


Figure 4. Degradation path of lithium battery

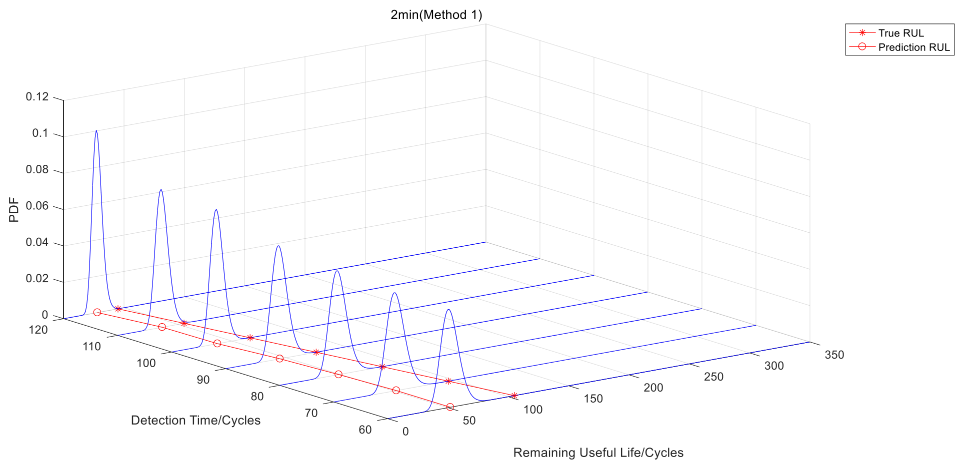
It can be seen from Fig 4 that the degradation paths of different recovery times are different, the life of lithium batteries with recovery effect is longer than the life of lithium batteries without recovery effects; the recovery time is 2 min, 5 min, 10 min. The longer the recovery time, the longer the life of the lithium battery. Here, the RUL prediction method in different cases is defined as follows: the RUL prediction methods that don't consider recovery effect in [27] and [28] is defined as method 1and method 2, the RUL prediction method proposed in this paper considering recovery effect is defined as method 3. For a lithium battery with a recovery time of 2 min, 5 min, and 10 min (recovery effect), respectively method 1, method 2 and method 3 were used to predict the RUL of the lithium battery, and the experimental results were compared. The parameters were estimated by the parameter estimation method in the fourth part.

5.4 Prediction of RUL in Four Recovery States

According to the degradation path of the lithium battery, as shown in Fig 4 (Unit2), under the influence of recovery time of 2 min, 5 min, and 10 min, the lithium battery exceeded the failure threshold after 165 cycle times, 184 cycle times, and 187 cycle times of charge and discharge, respectively. Therefore, it can be determined that the life of the lithium battery in the four recovery states is 165 cycle times, 184 cycle times, and 187 cycle times of charge and discharge. According to theorem 1 and theorem 2, under the influence of recovery, the life distribution and expected values at each detection time by using method 1 method 2 and method 3 are shown in Fig 5. In Fig 5, compared with the method 1 and the method 2, the life expectancy value predicted by the method and proposed in the method 3 is closer to the actual life, indicating that the prediction result of the method 3 is more accurate. Table 2 is the error comparison of the results predicted by the three methods. Fig 6 is the absolute errors of the three methods at each detection time. The experimental results of method 1, method 2 and method 3 can be better compared, and the feasibility of method 3 can be further analyzed.

Table 2. Comparison of error between three prediction methods.

Method	Recovery Time/min	Actual Life/ cycles	Predictive Life/ cycles	Absolute Error /cycles	Relative Error
Method 1	2	165	112	53	32.12%
Method 2	2	165	177	12	7.27%
Method 3	2	165	167	2	1.21%
Method 1	5	184	118	66	35.87%
Method 2	5	184	199	15	8.15%
Method 3	5	184	171	13	7.07%
Method 1	10	187	125	62	33.16%
Method 2	10	187	161	26	13.9%
Method 3	10	187	182	5	2.67%


2min(Method 1)
True RUL
Prediction RUL
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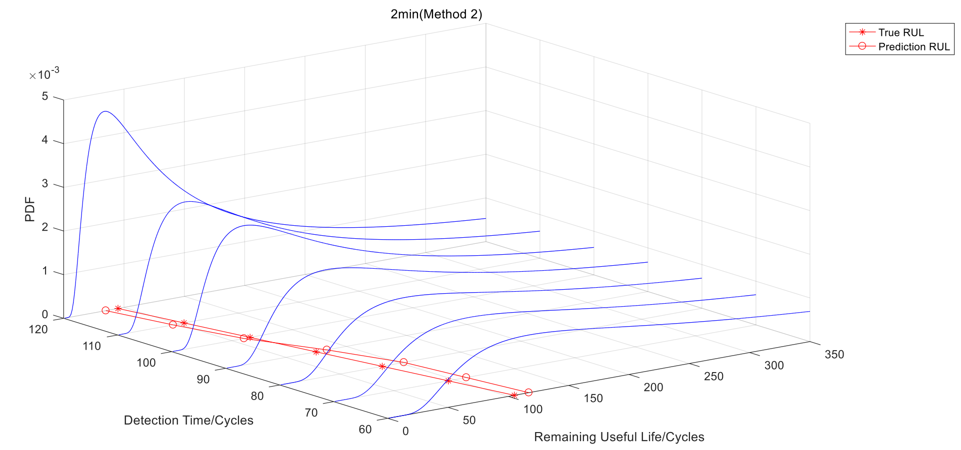
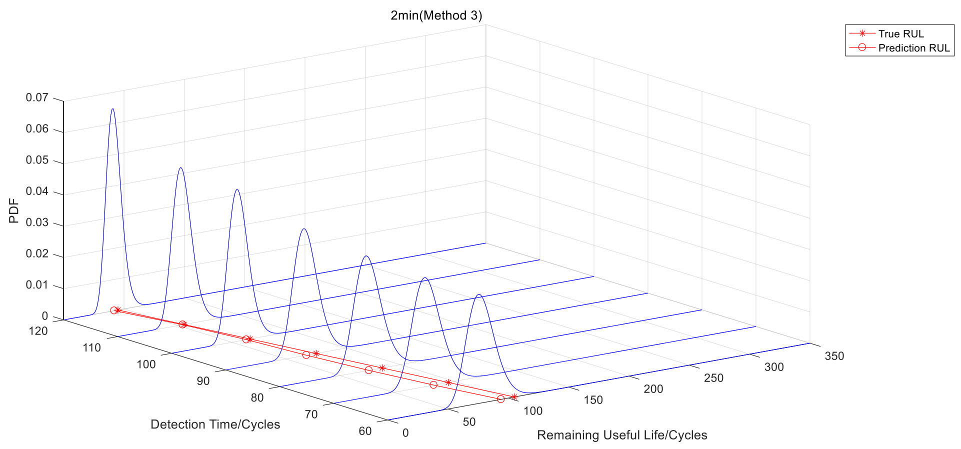
Detection Time/Cycles
Remaining Useful Life/Cycles


2min(Method 2)
True RUL
Prediction RUL
PDF
Detection Time/Cycles
Remaining Useful Life/Cycles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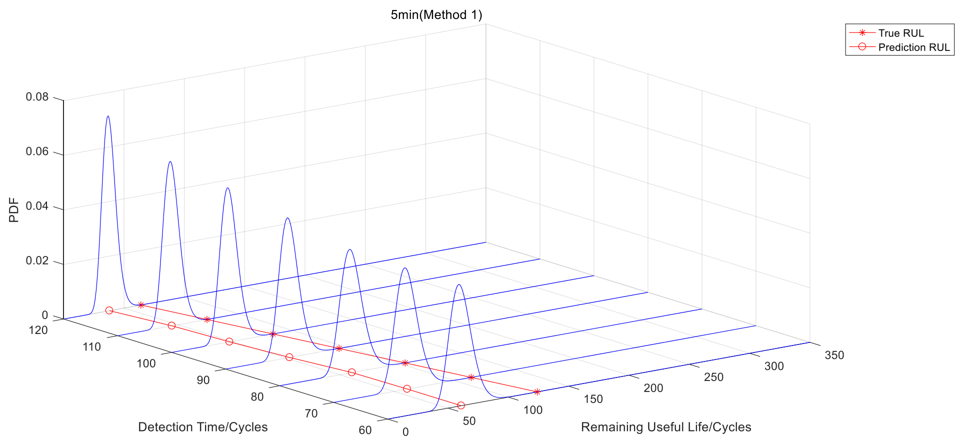
True RUL
Prediction RUL
PDF
Detection Time/Cycles
Remaining Useful Life/Cycles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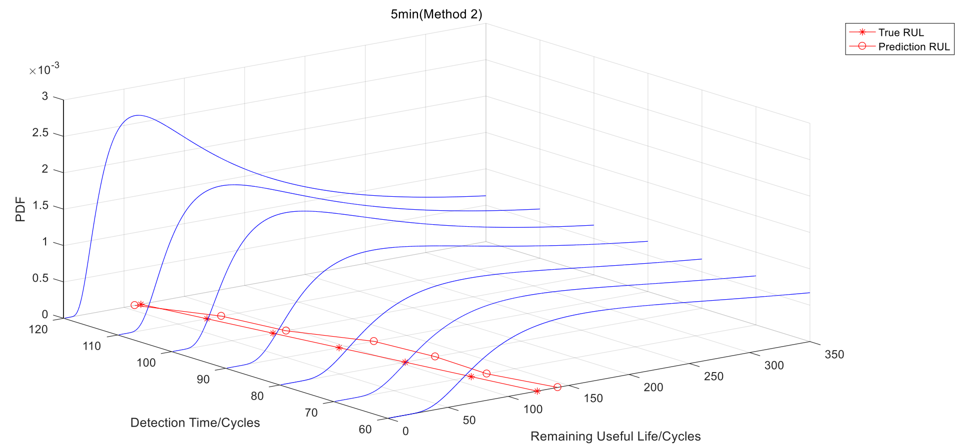
True RUL
Prediction RUL
PDF
Detection Time/Cycles
Remaining Useful Life/Cycles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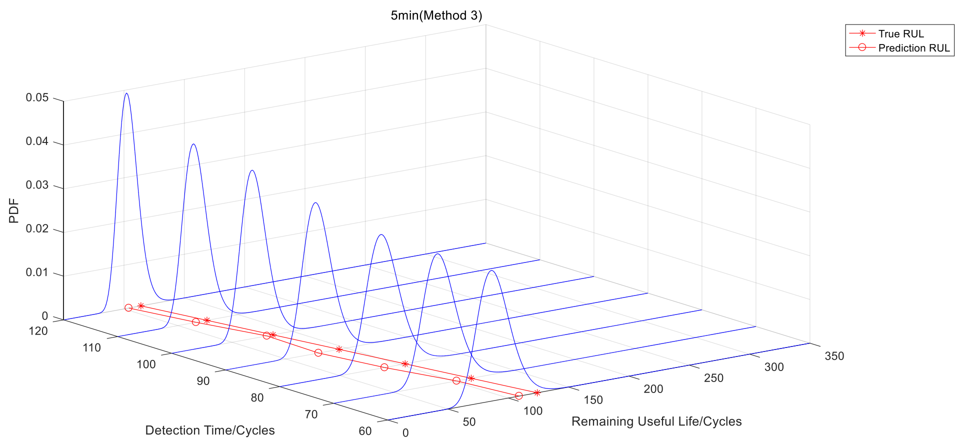
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True RUL
Prediction RUL
PDF
Detection Time/Cycles
Remaining Useful Life/Cycles

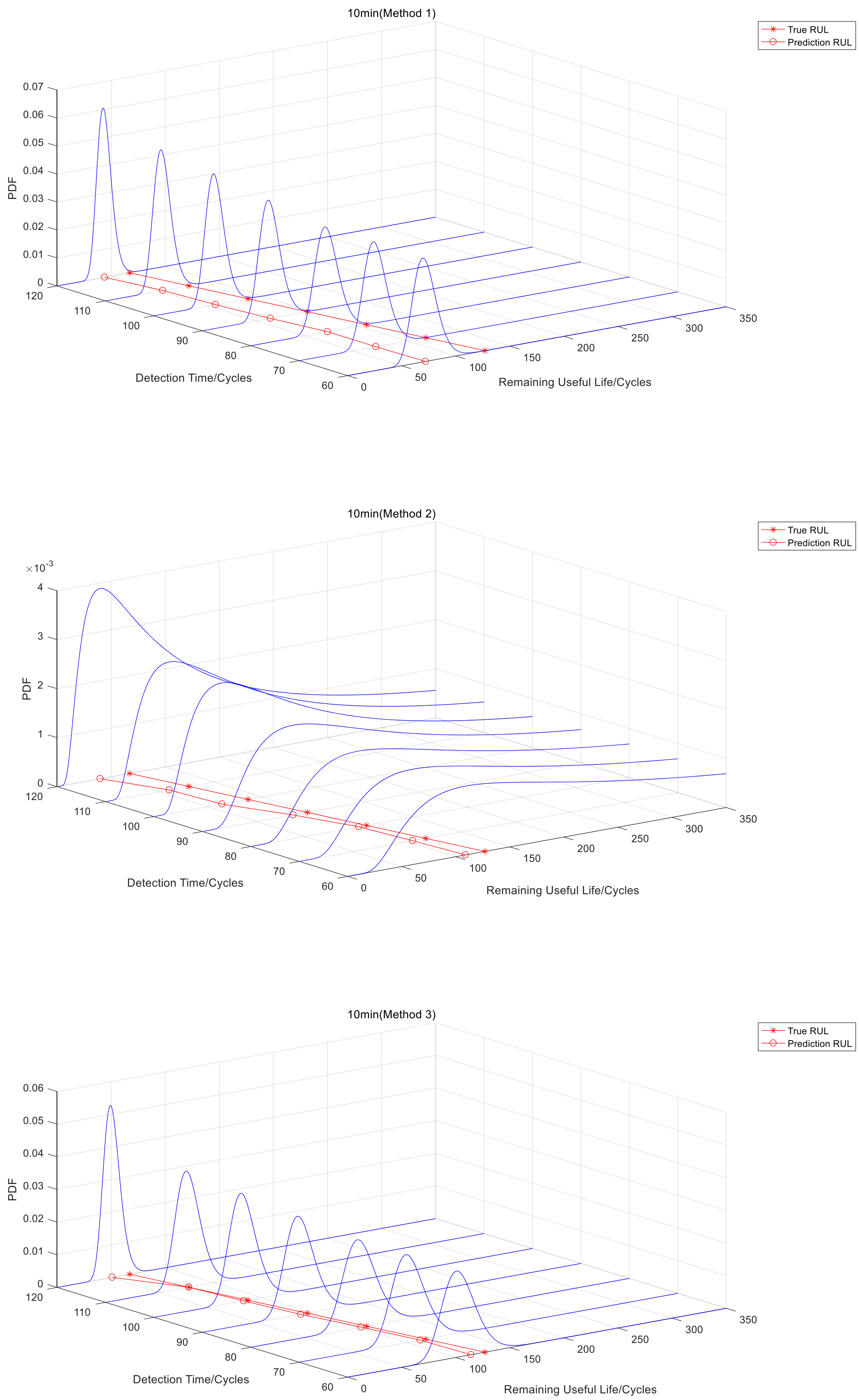

Figure 5. Comparison of remaining useful life probability density estimation results at each detection time

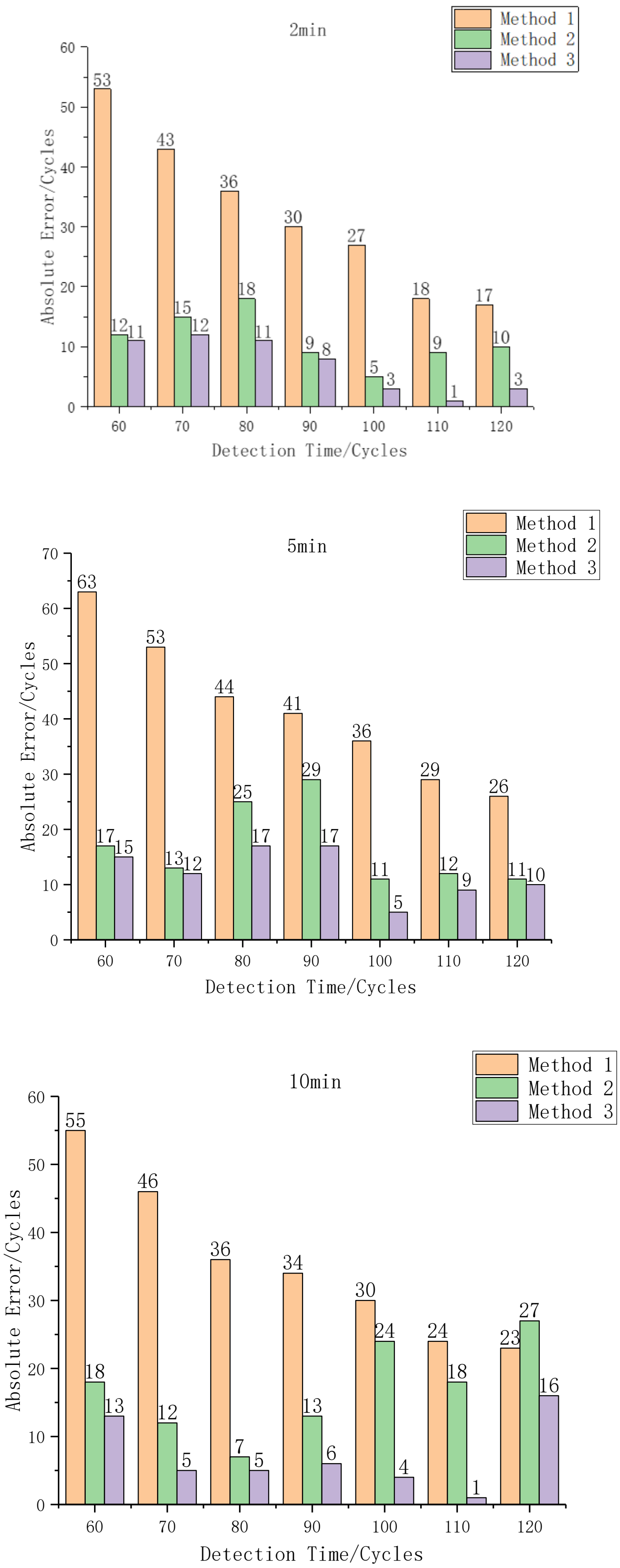

Figure 6. Absolute errors of the three methods at each detection time

By comparing Table 2, Figure 5 and Figure 6, the prediction results of Method 3 are significantly better than those of Method 1 and Method 2. Considering the recovery effect significantly improves the prediction accuracy of RUL.

6. CONCLUSION

Aiming at the recovery phenomenon in the process of lithium battery intermittent discharging , this paper proposes a lithium battery degradation modeling and RUL prediction method considering the recovery effect. (1) Based on the traditional Wiener process model, a lithium battery degradation model considering the recovery effect was established. (2) According to the degradation model, using the properties of the inverse Gaussian distribution, the analytical expression of the life expectancy of the lithium battery considering the recovery effect is theoretically derived. (3) Using the method of maximum likelihood estimation, the parameters of unknown parameters in the model are estimated. (4) Through the degradation data of 18650 lithium-ion battery, it is verified that the results obtained by the proposed method are better than those obtained without considering the recovery effect, which can improve the accuracy of life prediction. In further research, the RUL of the lithium battery can be estimated by the recovery phenomenon of the lithium battery caused by the stop discharge during the non-sustained discharge of the lithium battery, which will further improve accuracy of the RUL prediction of the lithium battery under the influence of recovery phenomenon.

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