

Influence of Physical Parameters and Soil Chemical Composition on Electrical Resistivity: A Guide for Geotechnical Soil Profiles

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The use of geophysical testing in geotechnical studies is sometimes looked at as a probable rather than certain approach when it comes to construct a precise subsurface soil profile. In order to obtain a detailed informative and comprehensive subsurface soil profile it will be required to conduct excessive borings and allow for huge costs. However, it can be possible to utilize the modern 2D and 3D electrical resistivity soil profiling techniques combined with few boreholes data to produce a reliable and correct subsurface profile, which can be used confidently by geotechnical engineers. In this study, the procedure of linking borehole information to the electrical resistivity ranges is set out. The predicted subsurface outcome for other electrical resistivity lines in the same area can be presented. This study considered only the soil density and soil type as prime factors. In order to validate the procedure further four investigation points were advanced and checked for matches and mismatches. Recorded errors were treated with a range of acceptance and rejection criteria. Further studies to introduce more adequate refinements are proposed. The resistivity of rocks and minerals can vary with the mobility, concentration, and ions dissociation. Dissolved chlorides and sulfates in water can result in different conductivities. The combined effect of some cations and anions together with some engineering parameters was studied for 15 samples of near surface sands and clays. Correlation studies and linear models using R statistical package were employed to highlight the influence of the chemical composition on the electrical resistivity results compared to other engineering properties and factors.

Keywords: Electrical resistivity; Metal conductivity; Clayey Soil; Chemical composition

1. INTRODUCTION

Electrical resistivity is a widely used geophysical approach in practice. It is found to be quick and reliable tool to classify and predict physical properties of materials based on the ease at which electric current can pass. Rock and minerals can be considered good conductors when the resistivity value is in the range of 10^{-8} to $1 \text{ } \Omega\text{m}$. Non-conductors can have a resistivity values in excess of $10^7 \text{ } \Omega\text{m}$. (Telford et al 1990)[1]. Minerals and rocks of intermediate conductivity normally vary between these two ranges. The electrical resistivity is influenced by many factors including moisture content, temperature, and compaction. The fluids chemistry, porosity and air voids are among these factors. Salinity and chemical composition can affect the measurements of the electric resistivity. Due to these many factors, electrical resistivity is rarely unique for a particular formation. Overlapping ranges are very common. In general, it can be observed that igneous rocks have the greatest resistivity values unlike sediment material, which is normally of low resistivity.

Table 1. Approximate ranges for electrical resistivity for selected materials.

Subsoil Material	Resistivity (Ωm)	
	Lower Bound	Upper Bound
Clays	1	100
Sand	4	800
Loam	5	50
Marl	3	70
Consolidated Shale	20	2000
Gravel/Conglomerate	2000	10^4
Coarse grained Sandstone	10	$9.6 \cdot 10^4$
Siltstone	1	$1.5 \cdot 10^4$
Sandstone	50	$8.0 \cdot 10^3$
Limestone	80	$1.0 \cdot 10^3$

The bound ranges stated in Table 1 are for guidance only and quoted differently by different researchers.

Ohm's law is giving the resistance R in Ohm as

$$R= V/I \tag{1}$$

Where V is voltaje in volts and I is current in amperes. Using four aligned electrodes placed at equal distances apart (a) known as Wenner array the apparent resistivity (ρ) is given by the formula:

$$\rho a = 2\pi RL \quad (2)$$

Where L is length measured in (m).

2. ELECTRICAL RESISTIVITY CORELATIONS

Many works were carried out to correlate the electrical resistivity values to some engineering parameters. The moisture content and the soil and moisture chemistry are among the most factors influencing the electrical resistivity values. The saline water of 20% concentration yields a resistivity average value of 0.05 Ωm where natural soil water can result in values of 9 to 100 Ωm (Telford et al 1990)[1]. This is mainly due to the chemical concentrations of dissolved ions, for example chloride and sulfate ions. It is not only the moisture content but also the chemistry of the pore fluid and soil material. The works of Syed Osman and Tuan Harith (2010)[2] investigated the trends of resistivity with moisture content, frictional angle, bulk density, and standard penetration tests. They found an increase in resistivity with the increase of frictional angle, bulk density, and SPT (standard penetration tests). The works were based on very limited number of samples. In fact, these three factors are directly proportional to each other and can all produce the same effect and influence. The increase in moisture is generally associated with decrease in the resistivity. Gil Lim Yoon and Jun Boum Park (2001)[3] showed that electrical resistivity in sandy soil depends largely on the water content and electrical properties of the pore water. The fines content (material passing sieve no. 200) is also found to have significant effect on the measured electrical resistivity. Kalinski and Kelly (1993)[4,5] demonstrated that electrical resistivity could have an excellent correlation to the volumetric soil moisture. Rhoades et al., (1989)[6] studied the accuracy of models that determine the soil salinity in the field from the measurements of electrical conductivity.

Fukue et al (1999)[7] claimed that greater electrical conductivity is associated with clays and fine textured soils than coarse textured. The pore size and distribution can be linked to the electrical resistivity. Samouëlian et al (2005)[8] stated that the porosity could be worked out using Archie's law for a saturated soil is given as:

$$\rho/\rho_{Bw} = a F^{-m} \quad (3)$$

ρ is the resistivity of the formation and ρ_{Bw} is resistivity of the pore water F is the porosity, a and m are constants.

The influence of moisture content on the electrical resistivity was studied extensively and was found that moist soils are associated with reduced electrical resistivity. Works of Goyal et al. (1996)[9], Zhou et al., (2001)[10], Binley et al. (2002)[11] and Abu-Hassanein (1996)[12,13]. confirms this generalized statement. In reality this is dependant on the pore and the soil particle chemistry. Kalinski and Kelly (1993)[4] found that soil solutions of the same concentration and different ionic composition might have different electrical resistivity due to differences in ion mobility. They claimed that ions within the solution like H^+ , OH^- , SO_4^{2-} , Na^+ , Cl^- ,...etc can have different conductivity values although of the same concentration.

Increase in temperature can result in decrease in soil electrical resistivity, Campbell et al (1948)[14]. It is always advisable to state the weather conditions and temperature when electrical resistivity measurements are taken.

In the current study, a trial is made to investigate the influence of the soil chemistry in the measurement of the electrical resistivity. The conductivity through rock and minerals can take different forms and can be either electronic as for cylindrical solids or electrolytic as for non-conductors with pores filled up with fluids. Dielectric conduction may also be considered for non-conductors. The dielectric constant varies with the amount of water present (Telford et al 1990)[1]. The dielectric constant for water at 20° C is 80.36 compared to values of 3.9 to 43 for soils and clays. The influence of a particular element cannot easily be estimated based on the laboratory measurements as conductivity can take different forms. The heterogeneous nature of the soil and interactions between different factors makes it rather difficult to assess certain electrical resistivity conditions. Therefore, the presence of a specific chemical component may have or may not have significant contribution in the measured electrical resistivity. Aluminum, calcium, iron, magnesium, silicon, sodium, and potassium ion concentrations were measured for fifteen soils and compared to the electrical resistivity. The sulfate content and chloride content expressed as percentage was also measured.

3. MATERIALS AND METHODS

2.1. Study area

The site used for this study is located in a semi-arid region in the kingdom of Saudi Arabia. A comprehensive geotechnical investigation consisting of borehole drilling works and electrical resistivity tests was carried out to investigate the subsurface formation.

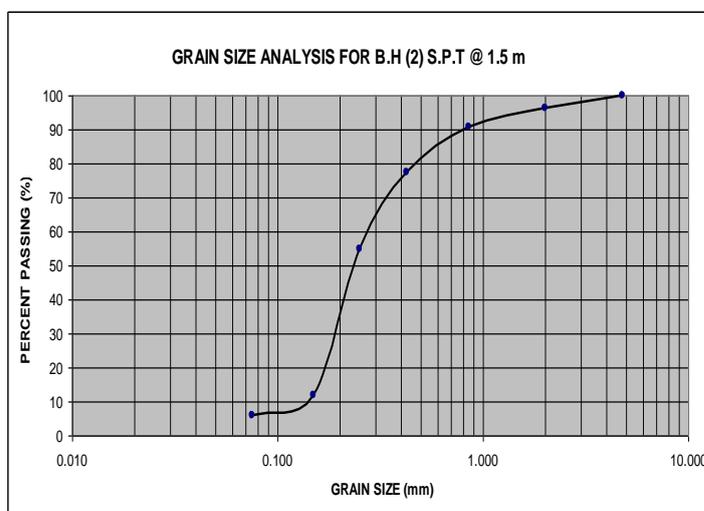


Figure 1. Grain size distribution for borehole 2 depth 1.5

The subsurface strata consist of top loose to medium dense sand followed by very dense sand intermixed with silt and occasional clayey material. Open test pits were also conducted to visualize and verify the soil conditions reported by using the electrical resistivity and soil boring. The fine material passing sieve number 200 included some gypsiferous silt material. The high concentration of gypsum was found to have adverse effect on the near surface material and some open cavities were formed. Below the top soil, weakly cemented sandstone is encountered.

The sandstone formation is found to continue to the maximum investigated depth of 25m below ground level. The scope of this research study was limited to the recognition of top soil type and density and the level of cemented sandstone rock. Three types of top soils are encountered; silty sand (loose), medium dense, very dense) sand (loose, medium dense, very dense) and sandstone (friable, weakly cemented, sound and intact)

Typical grain size distribution curves of top soils are shown on Figures 1.

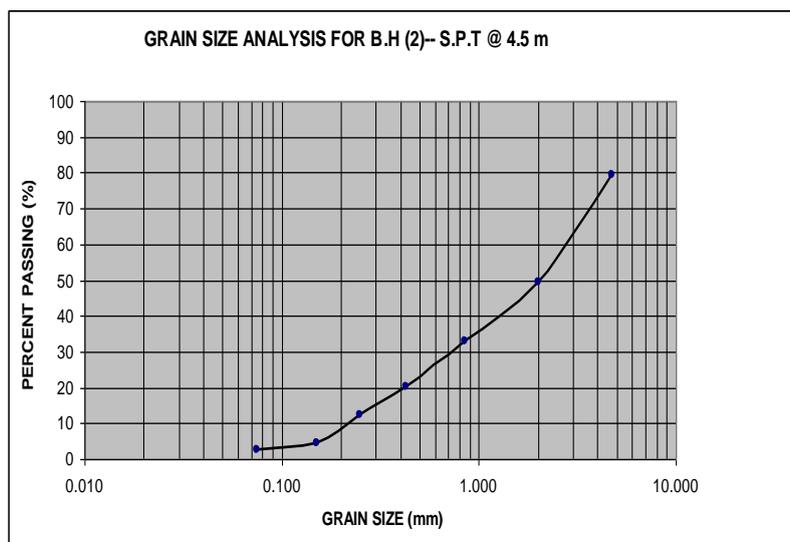


Figure 2. Grain size distribution for borehole 2 depth 4.5m

The above two charts show the sand material gradation at two different levels (1.5m and 4.5m). The material was classified in accordance with ASTM D2487 and found within groups of SW, and SP-SM.

Chemical tests carried out for sand samples taken from four (4) test pits indicated variations within close range for most of the elements, carbonates and sulfates. TP-1 indicated a rather high concentration in calcium and silicon.

The sandstone was encountered at depths varying from 6.5m to 9.0m. The sandstone cementation varied across the site from friable and weakly cemented to siliceous sound sandstone. The unconfined compressive strength varied from 12.07 to 19.09 MPa. No ground water was encountered within the investigated depths of 26m below ground level.

Table 2. Chemical test results for four sand soil samples

No.	Al ⁺⁺ (%)	Ca ⁺⁺ (%)	Fe ⁺⁺ (%)	Mg ⁺⁺ (%)	Si ⁴⁺ (%)	Na ⁺ (%)	K ⁺ (%)	Cl ⁻ (%)	So ₄ ⁻ (%)
1(sand)	1.6	19.6	0.5	1.1	34.4	9.6	1.9	0.2	4.5
2(sand)	1.7	22.2	0.8	1.5	21.3	6.5	0.8	0.33	4.6
3(sand)	4.4	13	2.3	1.6	27.6	3.5	0.6	0.37	4.7
4(sand)	6.6	21.4	3.7	2	25.6	2.7	0.5	0.41	4.8

The electrical resistivity equipment utilized in this study was Syscal R1. The system was supplied with 72 electrodes and capable of producing two-dimensional sections. The manufacturer of the system stated that the output current is automatically adjusted (automatic ranging) in order to optimize the input voltage values and thus provide high quality measurement. Such systems can be useful in many applications. Including salinity control, depth-to-rock determination and weathered bedrock mapping. It can also be used to determine shallow groundwater conditions (Aquifer depth and thickness). Electrodes spacing can be varied up to 10m. The system is also capable of transferring data to the computer and can be processed using specially tailored software (Res2DInv). The system can produce contour plots of apparent and true resistivity distribution. The system is designed to work in harsh climatic conditions including temperature range from -20 to 70 °C.



Figure 3. Syscal R1 Electrical Resistivity equipment.



Figure 4. The conventional drilling using a truck mounted rig.

4. TESTING PROGRAM

The fieldwork-testing program included performing six parallel electrical resistivity lines 9m apart. These were designed to cover an area, which was also covered by five geotechnical investigation points. Two boreholes and three test pits. Other boreholes and test pits were also conducted in the site but little far from the area covered by the electrical resistivity program. Drilling works were carried out using a Truck mounted drilling Rig (ACKER AD II) and test pits were performed using a hydraulic excavator. The standard penetration test was carried out at intervals of 1.5m in accordance with ASTM D1586. Figure (5) below shows the general locations of the investigation points.

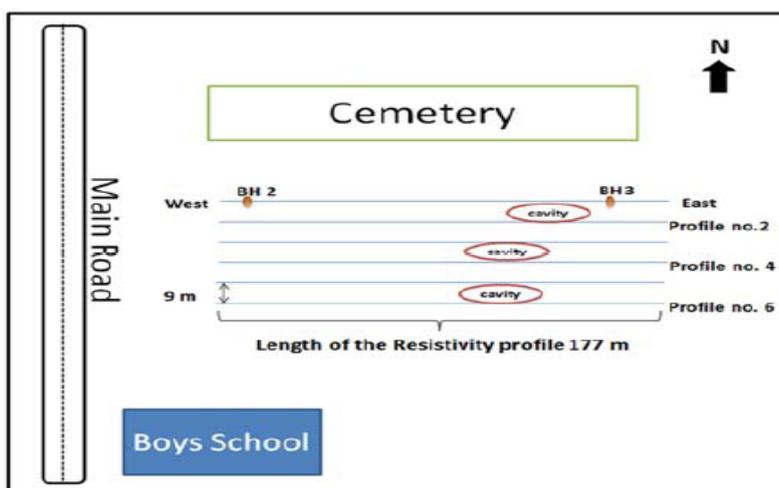


Figure 5. Distribution of electrical resistivity lines and investigation points

5. TEST RESULTS AND DISCUSSION

5.1 General

The electrical resistivity obtained from borehole No. 2 was noted and used as a base for the sand type and density level of the soil. The values obtained for the cemented sandstone were also taken as reference. The single borehole data was used to act as a bench mark for the resistivity values of the whole site. As mentioned above the electrical resistivity values for a certain type of soil is likely to vary greatly depending on several factors. We considered two approaches: In approach 1 we assumed the influence of the soil chemistry is negligible and in approach 2 we investigated the influence of the soil chemistry on electrical resistivity.

5.2 Approach 1: Negligible Influence of Soil Chemistry

For approach 1 the investigation considers some assumptions. These are as follows:

- The moisture content trends and changes within a vertical profile will be similar for the entire site.
- The soil chemistry and salinity of stratified layers will remain similar for the respective layers.

These assumptions can hold true for many sites and can allow straightforward interpretation of electrical resistivity results. A simplified comparison can lead the geotechnical engineer to forecast the density of sand formation across the whole site. Borehole 2 was considered as a control, for which formation encountered included medium dense sand, dense sand, very dense sand and cemented sandstone. The resistivity values reported for these types of soil were taken as reference and then used to classify other subsurface material for which electrical resistivity is known. The depth and extent of each layer can be established to a reasonably good accuracy. The works were carried out during hot summer and most of the near surface soils were semi-dry. Table 3 shows the electrical resistivity ranges measured for four types of formation mentioned for borehole number 2. The density of the formation is expressed in terms of the standard penetration test in which N (the number of blows of a 63.5 Kg hammer required to drive a 50mm diameter sampler tube 30cm) value determines the density level. N, (the number of blows in a standard penetration tests). The main goal of this work was to make the data obtained from the electrical resistivity more reliable and of a meaning to the geotechnical engineers. The electrical resistivity soil profile crossing both borehole 2 and borehole 3 is given in figure 6.

The drilling logs confirmed the predicted values of the electrical resistivity at boring number 3, which was selected as a validation point. It can be seen that at borehole 3 the electrical resistivity measured for top soils was in the range of 8.2 to 10.8 ohm m which was the value assigned to dense sand. No medium dense sand was expected. The Standard penetration tests carried out at borehole 3 confirmed this prediction. Very dense sand with electrical resistivity in excess of 16 ohm m and cemented sandstone with electrical resistivity in excess of 20 ohm m were also reported.

Table 3. Electrical resistivity ranges for borehole 2

Soil Profile	Standard Penetration Test (N blows)	Electrical Resistivity ωm
Dense fine grained sand with silt (0-2.0m)	35	10.8-16
Medium dense, medium grained sand with silt (2.0-3.5m)	12	8.2 -10.8
Medium dense, medium grained sand with silt (3.5-5.0m)	11	8.2 -10.8
Very dense, medium grained silty sand (5.0-6.5m)	>50	>16
Cemented sandstone	>50	>20

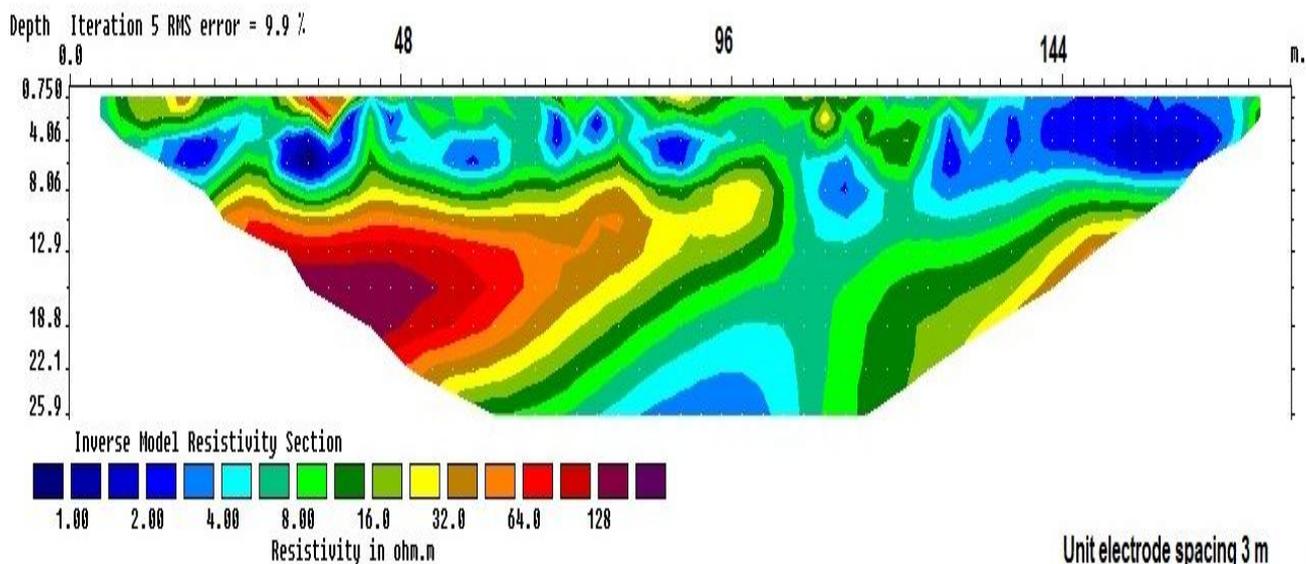


Figure 6. Electrical Resistivity profile for line Number 1 (typical software output).

Table 4. Electrical resistivity ranges for borehole 3.

Electrical Res. (ohm.m)	Predicted Profile	Measured Standard Test (N blows)
8.2-10.8	Dense sand with silt (0-2.0m)	41
8.2-10.8	Dense sand with silt (0-3.5m)	39
>16	Very dense silty sand (3.5-5.0m)	>50
>16	Very Dense silty sand (5-6.5m)	>50
>20	Cemented sandstone (>6.5)	NA

The validation using the test pit points was also confirmed for lines 2, 4 and 6. No standard penetration tests were carried out to check for the density but the type of soil down to three meters was found to coincide with the predicted formation.

The validation procedure adopted here is kept very simple and mainly based on limiting ranges. A more precise approach can be carried out if the control borehole is tested for different factors. For example establishing detailed density profile of top soils in relation to the measured electrical resistivity. This approach can give more weight to this procedure. Sites of heterogeneous nature can be investigated using more than one borehole. Additional boreholes may be located within areas of odd resistivity values.

There is no point for establishing correlation factors for all variables in a single site. The factor established for the density can work for the friction angle, compressive strength and other inter-related factors. As the values established in the literature for clays, silt and sand were given in wide ranges

they were hardly useful to evaluate a particular site. The assumptions made in each site shall be justified and reinforced by other physical measures in the laboratory. Soil classification and determination of fines content is very useful. The instrument used shall be regularly calibrated.

In order to validate the procedure further investigation points can be advanced and checked for matches and mismatches. Recorded errors can be treated with a range of acceptance and rejection criteria. Further studies were recommended to introduce better refinements and propose a generalized model that can be readily used by practicing geotechnical engineers.

Table 5. Electrical resistivity ranges for three test pits.

Location	Electrical Res. ωm	Predicted soil	Soil Description
TP-1	< 8	sand with silt (0-3.0m)	poorly graded sand with silt
TP-2	< 12	sand with silt (0-3.0m)	poorly graded sand with silt
TP-3	< 5	sand with silt (0-3.0m)	poorly graded sand with silt

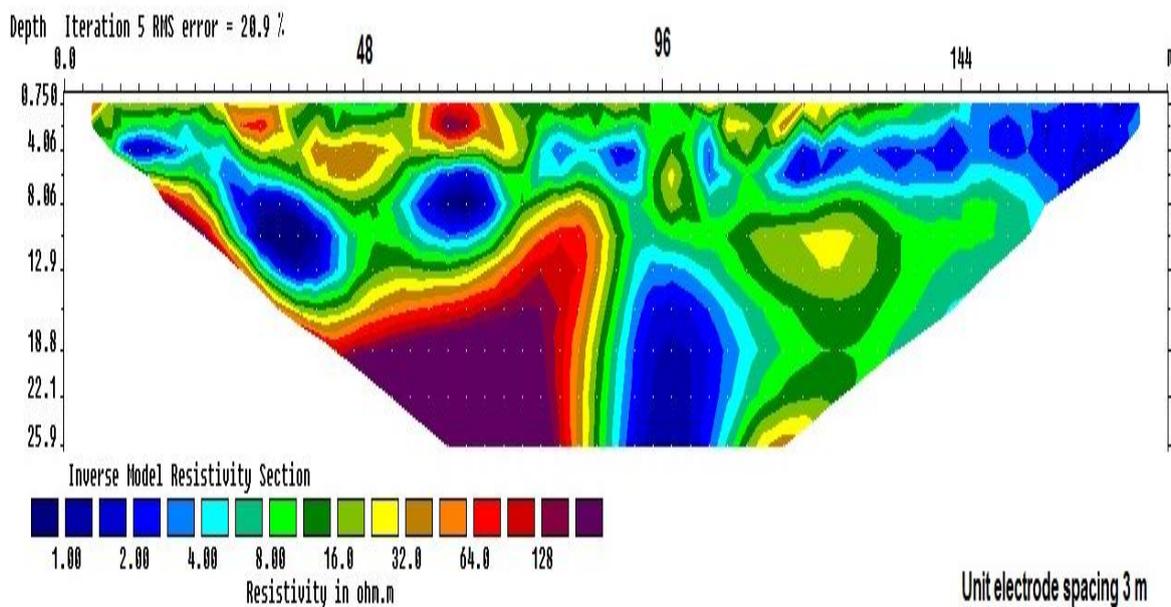


Figure 7. Electrical resistivity profile for line Number 5 (typical software output).

5.3 Approach 2: The Influence of Chemical Composition

In order to study the influence of chemical composition further 11 samples of different soil types were obtained from other three sites. The measured electrical resistivity and other soil parameters are listed. Fifteen samples were used to study weather links between the chemical composition and the electrical resistivity exist. Table 6 below gives details of investigated parameters:

Table 6. Electrical resistivity and soil chemical composition.

No.	Resist. (ρ)	Al ⁺⁺ (%)	Ca ⁺⁺ (%)	Fe ⁺⁺ (%)	Mg ⁺⁺ (%)	Si ⁴⁺ (%)	Na ⁺ (%)	K ⁺ (%)	Cl ⁻ (%)	So ₄ ⁻ (%)
1 (sand)	8	1.6	19.6	0.5	1.1	34.4	9.6	1.9	0.2	4.5
2 (sand)	12	1.7	22.2	0.8	1.5	21.3	6.5	0.8	0.33	4.6
3 (sand)	5	4.4	13	2.3	1.6	27.6	3.5	0.6	0.37	4.7
4 (sand)	8	6.6	21.4	3.7	2	25.6	2.7	0.5	0.41	4.8
5 (clay)	66	3.05	5	0.67	-	32.2	-	0.1	0.04	0.45
6 (clay)	52	2.48	5.4	0.95	-	31.3	-	0.13	0.01	0.45
7 (clay)	16.5	3.78	3	0.52	-	35	-	0.07	0.01	0.01
8 (clay)	15	5.2	14.4	4	0.6	33	14.4	6.5	0.06	0.3
9 (clay)	40	1.5	2.1	0.1	1.3	24.7	12.6	1.9	0.001	4.6
10 (c ss)	64	4.4	5.2	2.3	0.8	35.5	12.8	8.6	0.15	0.6
11 (c ss)	110	2.4	4.2	1.5	0.6	31.5	11.1	6.2	0.37	1.2
12 (clay)	71	1.4	21	1.37	-	2.34	-	0.05	0.02	0.01
13 (clay)	17	9.36	8	1.55	-	18.2	-	0.26	0.25	0.66
14 (clay)	61	10.7	17	1.6	-	10.9	-	0.07	0.07	0.12
15 (clay)	60	8.33	9	1.25	-	17.2	-	0.66	0.34	3.4

Note: C SS stands for cemented sandstone (ρ) is the soil resistivity in $\bar{w} m$.

In order to perform statistical analysis using R package software [15] the data was placed in an excel file together with all information on the standard penetration tests, fines content and moisture content. The correlation table for all the parameters was obtained.

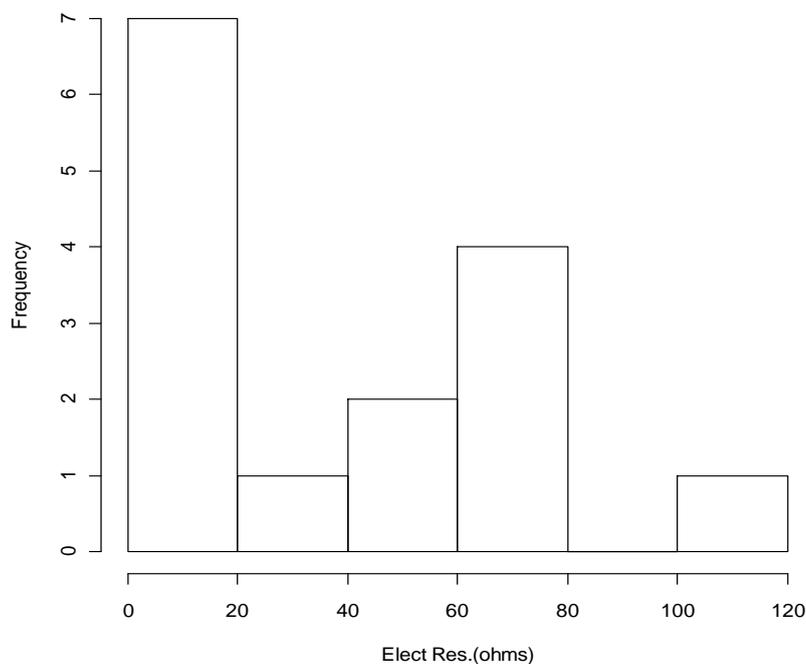


Figure 8. Frequency of electrical resistivity values

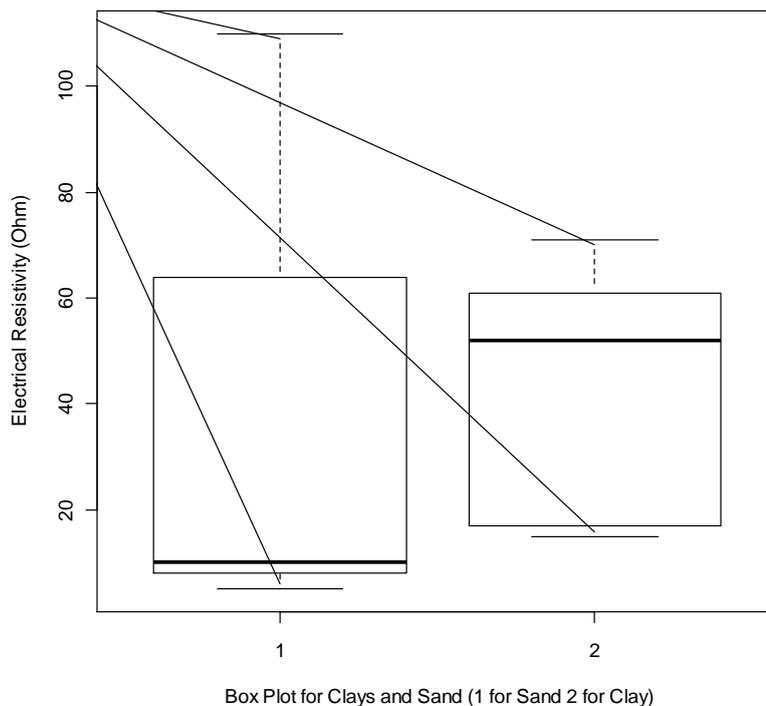


Figure 9. Box plot of electrical resistivity values

Correlation factors for electrical resistivity and various parameters:

Table 7. Correlation factors between electrical resistivity and tested parameters

Parameter	Correlation Factor	Parameter	Correlation Factor
Soil type	0.15758752	Mg	-0.52062115
density	0.52398440	Si	-0.17971371
finest	0.58915278	Na	0.40358136
Moisture content	0.46019109	K	0.29146014
Al	-0.07916953	Cl	-0.15341789
Ca	-0.38933044	So ₄	-0.47943702
Fe	-0.20826291		

The output of the statistical analysis using R package indicated that the strongest related parameters, placed in order, are: the finest content, magnesium, density, sulfate content, moisture content, sodium and calcium. It can be seen that the correlation with the soil type is very low compared to other factors. The high correlation of magnesium and sulfate content indicates that the influence of chemical composition shall not be neglected.

However, models based on this data were established to give the influence of some parameters on the electrical resistivity measurements.

Two models were considered; the first model assumes approach one in which soil chemistry was assumed is not significant and the second model considers the influence of chemical parameters. The first model in which the electrical resistivity was regressed on density, moisture content and fines resulted in an R^2 value of 0.434 while for the second model considering chemical parameters magnesium, sodium, calcium and sulfate content the R^2 increased to 0.587.

Approach 1 model is given by the equation:

$$\rho = -4.087 + 0.374 N + 0.468 fn - 0.888 mc \quad (4)$$

Where ρ is the electrical resistivity in Ωm , N is the standard Penetration Test value, fn is the fines content and mc is the moisture content.

Approach 2 model is given by the equation:

$$\rho = 129.191 + 0.296 N + 0.596 fn + 0.105 mc - 74.923 Mg + 6.402 So_4 - 6.550 Na - 0.853 Ca \quad (5)$$

Where Mg , So_4 , Na and Ca stands for the percentage of magnesium, sulfate content, sodium and calcium respectively.

It worse mentioning here that, the effect of confounding factors needs to be studied to refine the models and the assumption of using N value for different types of soil is very crude. Further refinements can be made in future works by considering large and diversified samples to arrive at models that describe the influence of chemical composition on the electrical resistivity more precisely.

The comparison between the obtained results and previous works indicates agreements in the general trends. For example the increase in the electrical resistivity with the increase in density supports the results of Syed Osman and Tuan Harith (2010) [2]. The decrease in electrical resistivity with the increase of moisture content is an agreement with the findings of many researchers (Telford et al 1990, Gil Lim Yoon and Jun Boum Park 2001) [1,3]. The current model investigated the combined effects of physical and chemical influence. It was found that magnesium, sulfate content, calcium and sodium have a significant influence on the measured electrical resistivity values.

6. CONCLUSIONS

The soil electrical resistivity can be used as an efficient tool to establish a reliable geotechnical profile. The internationally published figures for electrical resistivity of different formation are normally given in wide ranges. Most of them do not agree and these can result in excessive overlaps, which make it difficult to confirm or classify a material. Utilizing the information of a single borehole the overall site profile can be established. The soil density expressed as standard penetration test

number or measured laboratory density value can help recognizing the correlations to other factors. The procedure exercised in a site consisting of sand underlain by cemented sandstone yielded results with minimum mismatches. In order to refine the electrical resistivity, the influence of chemical composition is studied. It was found that magnesium, sulfate content, calcium and sodium have a significant influence on the measured electrical resistivity values. Preliminary models using R statistical package were developed. The results of this work is encouraging for future works and will help in using geophysical methods for quick, economical and yet reliable to provide useful geotechnical data.

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