

Improved Model Predictive Control for a Proton Exchange Membrane Fuel Cell

Liping Fan^{1,*}, Jun Zhang¹, Yi Liu², Xiaolin Shi¹

¹ Shenyang University of Chemical Technology, Shenyang, 110142, China

² North Heavy Industry Group Co., Ltd, Shenyang, 110141, China

*E-mail: fanliping@syuct.edu.cn

Received: 23 July 2012 / Accepted: 17 August 2012 / Published: 1 September 2012

Proton exchange membrane fuel cells are promising energy sources. Because of their time-change, uncertainty, strong-coupling and nonlinear characteristics, proton exchange membrane fuel cells present complex control challenges. Based on mathematical modelling and numerical simulations, two improved model predictive controllers which use Laguerre function and exponential data weighting are proposed for the proton exchange membrane fuel cell to realize constant power output. Simulation results show that the proposed model predictive controllers can give good control effects.

Keywords: Fuel cell; model predictive control; laguerre function; exponential data weighting

1. INTRODUCTION

Fuel cells are promising energy sources that produce electrical currents with almost null pollutant emissions. In the recent years there was an increasing interest in fuel cell technology. Although they were invented more than a century ago, they have received much attention in the last decade as good candidates for clean electricity generation both in stationary and automotive applications. Proton exchange membrane fuel cell (PEMFC) is one of the most interesting fuel cells types due to its low operating temperature, high efficiency and low electrolyte corrosion [1-3]. They are acknowledged as most valuable new alternative energy sources.

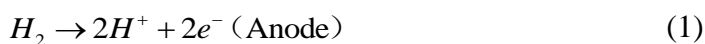
The performance of PEMFC, being important and getting more and more attention in recent years, is known to be influenced by many parameters such as operating temperatures both fuel cell and humidifiers, pressure, flow rates and relative humidity of fuel and oxidant gases. In order to improve fuel cell performances, it is essential to understand these parametric effects on the fuel cell operations

[4-5]. In many applications, keeping a fuel cell in a state of constant power output is necessary. So, maintaining a fuel cell system in correct operating conditions is necessary and it requires good control system. Model predictive control (MPC) is an optimization strategy for the control of constrained dynamic systems [6-7]. MPC uses multi-step prediction, rolling optimization and feedback correction control strategies [8], so it can not only give a good control effect and strong robustness, but also have an advantage of less demand on the accuracy of the model. It is an effective method to solve complex industrial process control [9-10]. However, some problems still need to be solved when using MPC in a real system. It is well know that the traditional approach of expanding the future control signal uses the forward shift operator to obtain the linear-in-the-parameters relation for predicted output in designing a MPC. In the case of rapid sampling, complicated process dynamics or high demands on closed-loop performance, satisfactory approximation of the control signal requires a very large number of forward shift operators, and leads to poorly numerically conditioned solutions and heavy computational load when implemented on-line. So, improved strategies are needed to solve these problems.

This paper is organized as follows. The mathematical model for a typical proton exchange membrane fuel cell is described in Section 2. Section 3 presents a brief description of designing improved MPC controllers for PEMFC. Simulation results are presented in section 4 to confirm the effectiveness and the applicability of the proposed method.

2. MATHEMATICAL MODEL OF PEM FUEL CELL

PEM fuel cell electrochemical process starts on the anode side where H_2 molecules are brought by flow plate channels. Anode catalyst divides hydrogen on protons H^+ that travel to cathode through membrane and electrons e^- that travel to cathode over external electrical circuit. At the cathode hydrogen protons H^+ and electrons e^- combine with oxygen O_2 by use of catalyst, to form water H_2O and heat. Described reactions can be expressed by the following equations [11-14]:



The output voltage V_{fc} of a single cell can be defined as the result of the following expression

$$V_{fc} = E_{nemst} - V_{act} - V_{ohmic} - V_{con} \quad (3)$$

in which E_{nemst} is the thermodynamic potential of the cell representing its reversible voltage, and

$$E_{\text{nemst}} = 1.229 - 0.85 \times 10^{-3} (T_{\text{fc}} - 298.15) + 4.31 \times 10^{-5} T_{\text{fc}} \left[\ln(P_{H_2}) + \frac{1}{2} \ln(P_{O_2}) \right] \quad (4)$$

where P_{H_2} and P_{O_2} (atm) are the hydrogen and oxygen pressures, respectively, and T_{fc} (K) is the operating temperature. V_{act} is the voltage drop due to the activation of the anode and the cathode:

$$V_{\text{act}} = 0.9514 - 3.12 \times 10^{-3} T_{\text{fc}} - 7.4 \times 10^{-5} T_{\text{fc}} \ln(C_{O_2}) + 1.87 \times 10^{-4} T_{\text{fc}} \ln(i) \quad (5)$$

where i (A) is the electrical current, and C_{O_2} is the oxygen concentration. V_{ohmic} is the ohmic voltage drop associated with the conduction of protons through the solid electrolyte, and electrons through the internal electronic resistance:

$$V_{\text{ohmic}} = i(R_M + R_C) \quad (6)$$

where R_C (Ω) is the contact resistance to electron flow, and R_M (Ω) is the resistance to proton transfer through the membrane, which can be described as

$$R_M = \frac{\rho_M \cdot l}{A} \quad (7)$$

$$\rho_M = \frac{181.6 \left[1 + 0.03 \left(\frac{i}{A} \right) + 0.062 \left(\frac{T_{\text{fc}}}{303} \right)^2 \left(\frac{i}{A} \right)^{2.5} \right]}{\left[\psi - 0.634 - 3 \left(\frac{i}{A} \right) \right] \exp \left[4.18 \left(\frac{T_{\text{fc}} - 303}{T_{\text{fc}}} \right) \right]}$$

where ρ_M ($\Omega \cdot \text{cm}$) is the membrane specific resistivity, l (cm) is the membrane thickness, A (cm^2) is the membrane active area, and ψ is a specific coefficient for every type of membrane; V_{con} represents the voltage drop resulting from the mass transportation effects, which affects the concentration of the reacting gases and can be described by the following expression:

$$V_{\text{con}} = -B \ln \left(1 - \frac{i}{i_{\text{max}}} \right) \quad (8)$$

where B (V) is a constant depending on the type of fuel cell, i_{max} is the maximum electrical current. The output power of the single fuel cell is

$$P_{\text{fc}} = V_{\text{fc}} i \quad (9)$$

Fig. 1 shows a generally accepted dynamic model of the PEM fuel cell, in which q_{O_2} is the input molar flow of hydrogen, q_{H_2} is the input molar flow of oxygen, K_{H_2} is the hydrogen valve molar

constant, and K_{O_2} is oxygen valve molar constant [15-16]. Based on the above described mathematical model, a Matlab/Simulink simulation model of the PEMFC can be set up [17]. Parameters of the Ballard Mark V fuel cell [18] are used in the simulation model.

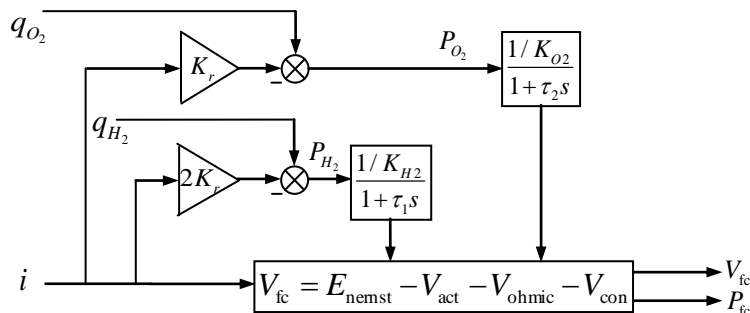


Figure 1. PEMFC dynamic model

3. DESIGN OF MPC AND IMPROVED MPC

Constant power sources are needed in some applications. So, keep fuel cells in constant power output state is necessary in such applications. In order to make the PEM fuel cell keep constant power output, MPC with laguerre and exponential data weighting for constant voltage output are designed and compared. The core technique in the design of discrete-time MPC is based on optimizing the future control trajectory, that is the difference of the control signal, $\Delta u(k)$. The traditional approach of expanding the future control signal uses the forward shift operator to obtain the linear-in-the-parameters relation for predicted output in designing a MPC. As a consequence, in the case of rapid sampling, complicated process dynamics or high demands on closed-loop performance, satisfactory approximation of the control signal requires a very large number of forward shift operators, and leads to poorly numerically conditioned solutions and heavy computational load when implemented on-line [19].

Laguerre function is famous for its orthogonality. Its expression is so simple that the computation load could get greatly reduced and it has a good effect on the parameterization presents a parsimonious description of the future control signal, hence reduces the number of parameters required in modelling the control trajectory. What's more, when the scaling factor is chosen at 0, the laguerre functions are equal to the expression of the traditional control variable, which is to say, Laguerre functions control strategy include the traditional MPC strategy. The z-transforms of the discrete-time laguerre networks can be shown as following [20]

$$\begin{aligned}
 \Gamma_1(z) &= \frac{\sqrt{1-a^2}}{1-az^{-1}} \\
 \Gamma_2(z) &= \frac{\sqrt{1-a^2}}{1-az^{-1}} \frac{z^{-1}-a}{1-az^{-1}} \\
 &\vdots \\
 \Gamma_N(z) &= \frac{\sqrt{1-a^2}}{1-az^{-1}} \left(\frac{z^{-1}-a}{1-az^{-1}}\right)^{N-1}
 \end{aligned} \tag{10}$$

where a is the pole of the discrete-time laguerre network and it is the scaling factor, and $0 \leq a < 1$ for stability of the network. Let $l_i(k)$ denote the inverse z -transform of $\Gamma_i(z, a)$, in which i is between 1 and N . This set of discrete-time laguerre functions are expressed in a vector form as $L(k)=[l_1(k), l_2(k), \dots, l_N(k)]^T$ and the laguerre sequences can be computed using the following state space model:

$$L(k+1) = HL(k) \tag{11}$$

where

$$H = \begin{bmatrix} a & 0 & 0 & \dots & 0 \\ \beta & a & 0 & \dots & 0 \\ -a\beta & \beta & a & \dots & 0 \\ a^2\beta & -a\beta & \beta & \dots & 0 \\ \vdots & & & & \\ (-1)^{N-2}a^{N-2}\beta & (-1)^{N-3}a^{N-3}\beta & \dots & \beta & a \end{bmatrix}; \quad L(0) = \sqrt{\beta} \begin{bmatrix} 1 \\ -a \\ a^2 \\ -a^3 \\ \vdots \\ (-1)^{N-1}a^{N-1} \end{bmatrix} \tag{12}$$

and $\beta = 1 - a^2$. At time k_i , the control trajectories $\Delta u(k_i), \Delta u(k_i + 1), \Delta u(k_i + 2), \dots, \Delta u(k_i + k), \dots$ is regarded as the impulse response of a stable dynamic system. Thus, a set of laguerre functions, $l_1(k), l_2(k), \dots, l_N(k)$ are used to capture this dynamic response with a set of laguerre coefficients that will be determined from the design process. More precisely, at an arbitrary future sample instant k

$$\Delta u(k_i + k) = \sum_{j=1}^N c_j(k_i) l_j(k) \tag{13}$$

with k_i being the initial time of the moving horizon window and k being the future sampling instant; N is the number of terms used in the expansion and $c_j, j=1, 2, \dots, N$, are the coefficients, and they are functions of the initial time of the moving horizon window k_i . Let $\eta = [c_1 c_2 \dots c_N]^T$ and $\Delta u(k_i + k) = L(k)^T \eta$ can be obtained. For a given discrete-time systems

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m u(k) \\ y(k) &= C_m x_m(k) \end{aligned} \tag{14}$$

Let $\Delta x_m(k) = [x_m(k) - x_m(k-1)]$, $\bar{y}(k) = C_m x_m(k)$, and choosing a new state variable $x(k) = [\Delta x_m(k) \ \bar{y}(k)]^T$, then the MPC state space model can be expressed as follows:

$$\begin{aligned} \begin{bmatrix} \Delta x_m(k+1) \\ \bar{y}(k+1) \end{bmatrix} &= \begin{bmatrix} A_m & O_m^T \\ C_m A_m & 1 \end{bmatrix} \begin{bmatrix} \Delta x_m(k) \\ \bar{y}(k) \end{bmatrix} + \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k) \\ y(k) &= [O_m \ 1] \begin{bmatrix} \Delta x_m(k) \\ \bar{y}(k) \end{bmatrix} \end{aligned} \tag{15}$$

where O_m is a zero matrix. The Eq. (15) can be written as:

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{16}$$

An improved predictive formulation in which $\Delta u(k_i + i)$ is replaced by $L(k_i)^T \eta$ can be shown as:

$$\begin{aligned} x(k_i + m | k_i) &= A^m x(k_i) + \sum_{i=0}^{m-1} A^{m-i-1} BL(i)^T \eta \\ y(k_i + m | k_i) &= CA^m x(k_i) + \sum_{i=0}^{m-1} CA^{m-i-1} BL(i)^T \eta \end{aligned} \tag{17}$$

where $x(k_i + m | k_i)$ and $y(k_i + m | k_i)$ are the predicted state variables and the predicted output at k_i+m with given current plant information $x(k_i)$. Then we get a new expression of the cost function

$$J = \eta^T \Omega \eta + 2\eta^T \Psi x(k_i) \tag{18}$$

where $\Omega = (\sum_{m=1}^{N_p} \phi(m)Q\phi(m)^T + R_L)$, $\Psi = (\sum_{m=1}^{N_p} \phi(m)QA^m)$, R_L is a designed matrix; and $\phi(m) = \sum_{i=0}^{m-1} A^{m-i-1} BL(i)^T$ with $Q = C^T C$.

To find the minimum of Eq. (18) without constraints, we assume that Ω^{-1} is existent. Then the optimal solution of the parameter vector η can be obtained by letting the partial derivative of the cost function approach to zero.

It is noted that in the finite prediction horizon, the condition number of the matrix Ω increases as the prediction horizon N_p increases. When there is an integrator in the system matrix A , the norms of the matrix power $\|A_m\|$ do not decay to zero as m increases. Thus, the magnitudes of the elements in Ω

increase as the prediction horizon N_p increases. Hence, if the prediction horizon N_p is large, a numerical conditioning problem occurs. This numerical problem becomes severe when the plant model itself is unstable, or when the dimension of the matrix A is large. An exponentially weighted moving horizon window can convert the numerically ill-conditioned matrix Ω into a numerically well-conditioned in the presence of a large prediction horizon and could deal with great changes of the plant which originally get serious problems.

Define the sequence of exponentially weighted incremental control as

$$\Delta \hat{U} = [\alpha^{-0} \Delta u(k_i), \alpha^{-1} \Delta u(k_i + 1), \alpha^{-2} \Delta u(k_i + 2), \dots, \alpha^{-N_p} \Delta u(k_i + N_p)]^T \quad (19)$$

where $\alpha > 1$ is used to scale the eigenvalues of matrix A , and the exponentially weighted state variable is denoted as

$$\hat{X} = [\alpha^{-1} x(k_i + 1|k_i), \alpha^{-2} x(k_i + 2|k_i), \alpha^{-3} x(k_i + 3|k_i), \dots, \alpha^{-N_p} x(k_i + N_p|k_i)]^T \quad (20)$$

By using these exponentially weighted variables, the exponentially weighted cost function is expressed in terms of the transformed variables. The result is summarized by the theorem below:

$$\hat{J} = \sum_{j=1}^{N_p} \hat{x}(k_i + j|k_i)^T Q \hat{x}(k_i + j|k_i) + \sum_{j=0}^{N_p} \Delta \hat{u}(k_i + j)^T R \Delta \hat{u}(k_i + j) \quad (21)$$

where Q and R are weight matrices, $\hat{x}(k_i + j|k_i)$ and $\Delta \hat{u}(k_i + j)$ are governed by the following difference equation:

$$\hat{x}(k_i + j + 1|k_i) = \frac{A}{\alpha} \hat{x}(k_i + j|k_i) + \frac{B}{\alpha} \Delta \hat{u}(k_i + j) \quad (22)$$

In this research, when the plant is running under the traditional MPC, the predictive horizon is 20, the control horizon is 5, and the output weight is a unit matrix. When the plant is running under the MPC with laguerre functions substituting the control variables and under the MPC with exponentially weighted corrected, the two core factors are: a is 0.8 and N is 1.5, the predictive horizon is 48, the weighting $Q = C^T * C$, and $R=0.3$; what's more, α is chosen as 1.25 in the MPC with exponentially weighted corrected control strategy. The sampling time of the three control strategies is 0.1s. The control variables change in the range of [0, 10] when oxygen flow is chosen to adjust and [0, 20] when hydrogen flow is chosen to adjust, and the incremental of control variables ranges from -2 to 2. The output constrains are ignored in this paper. So far, all the parameters are all given.

4. RESULTS AND DISCUSSION

In order to verify the control effect of these designed controllers, simulation operation of the

fuel cell system with these various controllers is carried out. For the purpose of designing MPC controller, Least Square Technique is used to identify the state space model of PEM fuel cell described in Eq. (14), and the identification solutions of the coefficient matrices are

$$\begin{aligned}
 A_m &= \begin{bmatrix} 0.9226 & -0.0153 \\ 0.0961 & 0.9992 \end{bmatrix} \\
 B_m &= [0.0961; 0.0049] \\
 C_m &= [0 \quad 0.0378] \\
 D_m &= 0
 \end{aligned}
 \tag{23}$$

According to the principle of MPC, the parameter matrices corresponding to Eq. (16) can be derived as

$$\begin{aligned}
 A &= \begin{bmatrix} 1.9218 & -0.9233 & 0.0002 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.9218 & -0.9233 & 0.0002 & 1 \end{bmatrix}, B = \begin{bmatrix} 0.0002 \\ 0 \\ 1 \\ 0.0002 \end{bmatrix} \\
 C &= [0 \ 0 \ 0 \ 1], \quad D = 0
 \end{aligned}
 \tag{24}$$

Three control strategies are designed and compared in this paper, including traditional MPC with reduced horizon control, improved MPC with Laguerre functions and improved MPC with exponential data weighting. Meanwhile, these three controllers are designed for two control schemes respectively, one is control the output power by adjusting the hydrogen flow, and the other is control the output power by adjusting the oxygen flow.

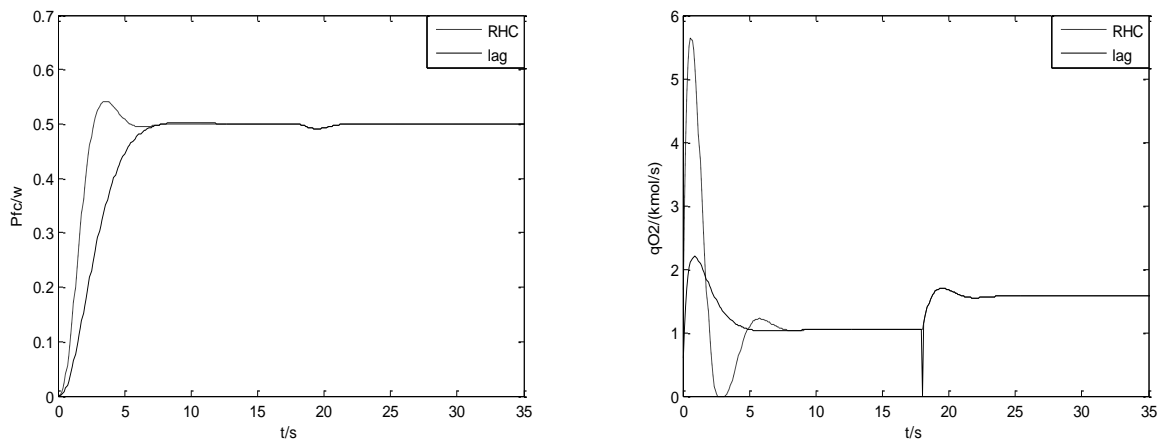


Figure 2. Traditional MPC and Laguerre based MPC with oxygen flow as the control variable

The reference setting output power of the fuel cell is 0.5W. In order to test the disturbance rejection ability of the designed controllers, load is set to change from 5Ω to 10Ω at the time of 18s.

The main parameters of the fuel cell model used in these operations are $B=0.016V$, $A=50.6cm^2$, $T_{fc}=343k$, $R_c=0.0003\Omega$, $l=0.0178cm$, $\psi=23$.

Figure 2 to Figure 5 report the simulation operation results under these dissimilar control schemes. In these figures, RHC stands for the traditional MPC strategy, lag stands for improved strategy with Laguerre functions and exp stands for the improved strategy with exponential data weighting. Multiple comparisons are shown in these figures.

It can be seen from these figures that the control effects are getting more and more well through step-by-step improvement schemes. The traditional MPC can make the system reach an elementary control objective roughly, but it cause big overshoot and long regulating time. The improved MPC with Laguerre functions or exponential data weighting can give better control effects. Both oxygen flow and hydrogen flow can be used as operating variable, and the control effects are similar, but the tracking time caused by adjusting oxygen flow is a bit shorter than that of adjusting hydrogen flow. When load disturbance occur in the system, the improved MPC can make the system return the given steady state rapidly after a short fluctuation. Analyzing by synthesis of steady state error, rapidity and overshoot, the improved MPC with exponential data weighting can give more desirable control effect than the other two control schemes, and the output power can keep in a given value well.

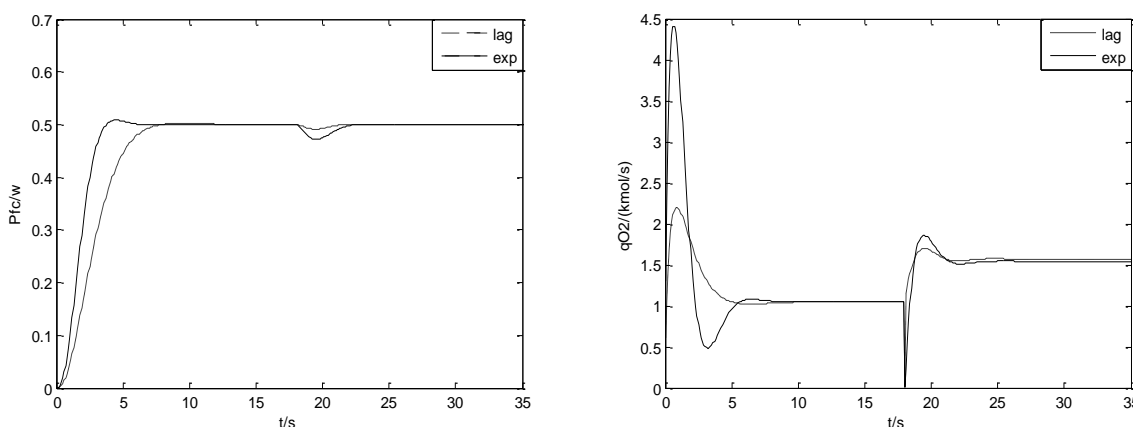


Figure 3. Improved MPC based on Laguerre function and exponential data weighting adjusting oxygen flow

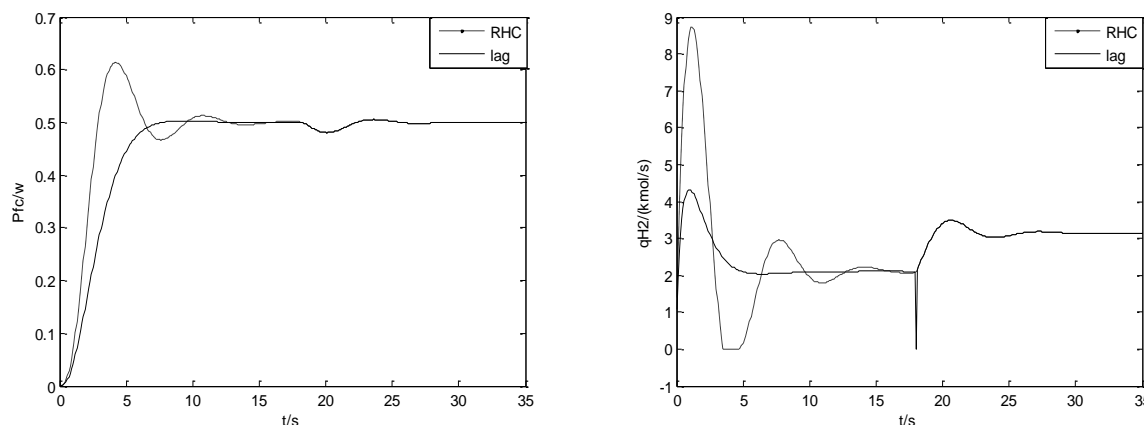


Figure 4. Traditional MPC and Laguerre based MPC with hydrogen flow as the control variable

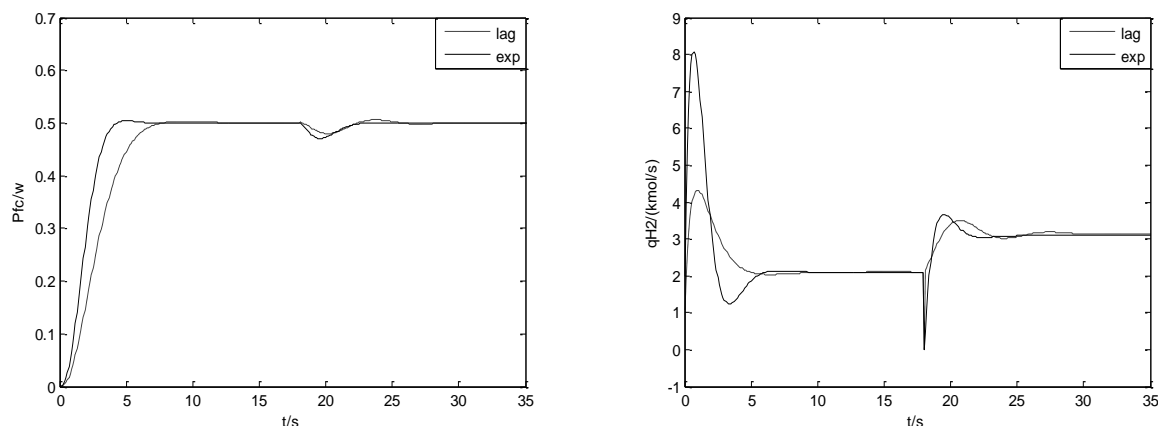


Figure 5. Improved MPC based on Laguerre function and exponential data weighting adjusting hydrogen flow

By comparing the results with those results published by other authors [13, 16, 21], it also can be seen that the method presented in this paper can realize outputting a constant power while the other references focused on keeping a constant output voltage. Further, compared with the former method in the same situation of controlling constant power [22], the method proposed in this paper has better control precision and adaptability.

5. CONCLUSIONS

Fuel cells which are required to output constant powers need good power control systems. By using improved MPC controller, the PEM fuel cell can not only have fast response characteristic, but also have good steady-state behavior and strong robustness. The suitable model predictive control scheme can get satisfactory results in tracking a given power and make the fuel cell output a required constant power.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China under Grant 61143007, and the National Key Technology Research and Development Program of China under Grant 2012BAF09B01.

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